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This publication contains sixteen abstracts of papers presented at the Research Reporting Sessions of the National Council of Teachers of Mathematics (NCTM) Annual Meeting. The investigations reported by Anthony, Creswell, Higgins, and Weise focus on curriculum and classroom innovations in the school mathematics program. Investigations by Gibbons, Pearce, Rector, and Wells relate to instruction and methods for teaching mathematical concepts. Research presented by Carry, Fey, Miller, and Vigilante is concerned with certain logical inference patterns and reasoning abilities in students. Bell and Gorth report the results of a survey of mathematics teachers with respect to current issues and a new design for evaluation in mathematics. Research on behavioral objectives for a seventh grade mathematics and for a freshman calculus course is reported by Bierden and Picard. (RP)

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PAPERS
for the
RESEARCH REPORTING SECTIONS
# of the
Forty-seventh Annual Meeting

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

at the

Leamington Hotel
Minneapolis, Minnesota

April 24-26, 1969

Sponsored by the

Research Advisory Committee
of the National Council of Teachers of Mathematics

Thomas A. Romberg, Chairman
Robert Bechtel
Edward Carrol
Edgar Howell
Thomas Kieren
Patrick Suppes

SE006 657

Introduction

For the second year, The Research Advisory Committee is sponsoring reports of research in sessions of the NCTM Annual Meeting. Both years the response has been overwhelming. Many very good papers have been submitted. The papers selected represent only a small subset of the papers which were submitted. In addition, this year it was decided to publish long abstracts of all the papers that would be reported at each of the sections.

These papers should be read and reviewed prior to the program so that the individual reporting can discuss the implications and points of interest of his work to a knowledgeable audience. There are four reporting sections or sixteen papers selected for this years presentation. One alternate paper has also been included. We think the idea of research reports and this procedure for reporting results of research has promise and invite your comments.

For the Research Committee
Thomas A. Romberg, Chairman

RESEARCH REPORT SESSION I

THURSDAY

11:00 a.m.--12:00 noon Jackson/Lincoln/Roosevelt (L)

Presider: Thomas Kieren, University of Alberta, Edmonton, Alberta

William P. Gorth, Stanford University, Stanford, California

A New Design for Evaluation in Mathematics: Longitudinal Comprehensive Achievement Monitoring

James Bierden, The University of Michigan, Ann Arbor, Michigan

Behavioral Objectives and Flexible Grouping in Seventh Grade Mathematics

James T. Fey, Columbia University, New York City, New York

Patterns of Verbal Communication in Mathematics

Max S. Bell, University of Chicago, Chicago, Illinois

A Survey of High School Mathematics Teacher's Backgrounds and Their Opinions and Priorities with Respect to Current Issues in Mathematics Education

A New Design for Evaluation in Mathematics Education: Longitudinal Comprehensive Achievement Monitoring

William P. Gorth, Stanford Center for
Research and Development in Teaching

and

Dwight W. Allen, The University of Massachusetts

The authors will describe a new design for longitudinal comprehensive achievement monitoring. The design has been developed under the sponsorship of a four-year grant from the Charles F. Kettering Foundation. It has been in operation in five high-school mathematics courses for almost two years. The components of the design which distinguish it from the usual classroom achievement testing are: (a) a model of school learning to direct the choice of variables to measure, (b) a comprehensive set of course objectives defined in behavioral terms, (c) a longitudinal scheduling of testing, and (d) a complete system to process data and report results to the teacher and individual students. The usual classroom achievement testing does not have a model of school learning, schedules tests for immediately after instructional treatment, uses a narrow set of generally poorly defined objectives and makes only a crude analysis of results.

The model. Several recent researchers have attempted to model school learning. A semi-quantitative model (Carroll, 1967) appears particularly useful as a starting point in choosing variables to measure to adequately describe school learning, but should be expanded.

Our model incorporates student and instructional variables. First, our model includes a measure of student motivation rather than of perserverance, which Carroll uses. Second, although a student may be motivated to perform a particular task, his anxiety at entering the task situation or being evaluated may interfere with several of the skills which are necessary for performing the task. The effect of anxiety on complex learning has been investigated by Spielberger and will be included in our model. Third, sociological variables are included and appear particularly important in public schools which enroll students from a broad range of family backgrounds. Fourth, the variables of rate of learning and initial achievement correlate very highly with achievement output, but have been found in preliminary studies to be independent of one another. Finally, the instructional variables include measures of the teachers, treatment characteristics, and time.

Comprehensive objectives. It has only been in recent years that teachers have begun to define their objectives in behavioral terms. Authors like Mager have written persuasively in favor of specifying instructional objectives in terms of observable student behaviors. The teacher is expected to define the objectives of his entire course before it begins in terms of observable student behavior by composing questions which measure acceptable levels of student performance for the objectives he set for his course.

Longitudinal testing schedule. The third distinguishing aspect of the comprehensive achievement monitoring is the longitudinal consideration of students' achievement on each course objective. In usual classroom testing a test of achievement is given immediately after an objective or set of objectives has been presented by the teacher. The test usually includes items which measure only the objectives taught since the last testing. Therefore, the teacher has available estimates of student achievement only on the objectives he has just taught. The usual situation contrasts with that of comprehensive achievement monitoring where an estimate of student achievement on each of the objectives is available at each testing period. Therefore, teachers can make statements about a student's pre-instruction, post-instruction, and retention of material as well as rate of learning. For example, if objectives one through ten are taught consecutively, and achievement is monitored after each is taught, then at time four, just after objective four has been taught, the estimate of student achievement on objective four is an immediate posttreatment achievement. The estimates made at the same time for objectives one through three are retention measures and those of objectives five through ten are pre-instruction measures.

The system. The system for achievement monitoring includes the parts: (a) model of the parameters, (b) focus of the evaluation, (c) resources for the evaluation, (d) collection of the data, (e) organization of the data, (f) analysis of the data, and (g) report of the analysis. Computer programs have been developed to handle the large tasks of analysis and reporting results. Other components of the system have been designed to operate at several levels, i.e., both with or without the computer.

Implications. The new design for evaluation in mathematics education provides longitudinal, comprehensive information on the achievement of individual students and groups of students on the performance objectives of the course. It is able to provide information on achievement on a standard scale for the course across time (a) before instruction, (b) immediately after instruction, and (c) a long time after instruction.

The data collected by the system could be used in several ways. First, course improvements could be made based upon the model of school learning. High entry achievement, low post-instruction achievement, or forgetting by the students in a course would each justify modifications in the course. Second, individual student school learning patterns could be plotted and decreases, plateaus or sharp rises in achievement would suggest different activities for the students. Finally, if alternate instructional treatments were administered, each to part of the class, the achievement patterns of the parts would display any differences in achievement between them immediately after or several weeks after the treatment. Therefore, the system could be used in empirical research in teaching.

Behavioral Objectives and Flexible Grouping In Seventh Grade Mathematics

James Bierden, The University of Michigan

The purpose of the study reported in this paper was to explore and develop a form of classroom management designed to make better provision for individual differences among students in a seventh grade mathematics course. This study was an outgrowth of an awareness of and a concern for classroom management problems resulting from individual differences of students of mathematics, especially among heterogeneously grouped junior high school students. As educators have become more aware of these differences, they have also become aware of the problems associated with providing for them within the milieu of formal education. Although much has been written about individual differences in the literature of education, the problem still remains a major concern for most classroom teachers.

The two major variations from normal classroom procedures involved in the study were (1) the classroom use of detailed behavioral objectives related to the content of a seventh grade mathematics course and (2) a form of classroom management using a combination of whole class instruction and flexible intra-class grouping based on achievement of the objectives.

Five major criteria were identified from the literature as being important in the effective use of flexible intra-class grouping: (1) the need for some combination of subjective and objective standards as the basis for establishing groups; (2) the need to maintain flexibility on the grouping procedures; (3) the need for proper selection of content, methods and materials for the groups; (4) the need for methods that preserve class unity; and (5) the need to minimize the loss of self respect of the "slow" pupils.

The decision to use behavioral objectives in the experimental treatment was influenced by three major reasons that have been given in the literature for the specification of educational objectives in behavioral terms:

(1) Behavioral objectives clearly communicate the terminal behaviors to be developed and guide the selection of content and instructional procedures;

(2) Behavioral objectives facilitate the evaluation of the extent to which students individually and collectively are achieving the objectives of an instructional program;

(3) Behavioral objectives provide advantages to the student by providing goals, the means to assess his progress, and the means for organizing his learning.

Two seventh grade classes of the University School, The University of Michigan were taught by the author using the classroom management procedure during the 1967-68 school year. Detailed behavioral objectives were written at three levels--Basic, Intermediate, and Advanced--for each topic covered in the course.

In its final form, the management procedure for each topic included the following steps:

(1) initial instruction of the topic to the whole class designed to meet the intermediate level objectives;

(2) testing the students on these objectives;

(3) assigning each student to a basic, intermediate or advanced group depending on his achievement of the objectives on the test;

(4) small group and individual instruction aimed at students' unachieved objectives;

(5) a second test on these unachieved objectives.

Student variables measured in the study included IQ (California Test of Mental Maturity), computational skills (arithmetic subtest of the California Achievement Test), knowledge of mathematical concepts (SMSG Mathematics Inventory), attitudes twoard mathematics (Aiken and Dreger Opinionnaire), test anxiety (Test Anxiety Scale for Children), flexibility of intra-class group membership, and reactions toward the experimental procedures.

The data collected in the study were analyzed using a variety of statistical procedures. The results of the analyses are summarized in the following findings.

1. The experimental students showed significant flexibility in their group membership ($p < .01$).

2. A linear regression analysis yielded significant multiple correlation coefficients ($p < .01$) indicating that a linear combination of measurements of IQ, computational skills, knowledge of mathematical

concepts and attitudes toward mathematics accounted for 74 per cent of the variance in group membership.

3. Four subgroups of students--homogeneous with respect to IQ, mathematical achievement, attitudes and group membership--were identified using a cluster analysis computer program. Each of these subgroups exhibited different interactions with the experimental treatment in terms of their mathematical performance, attitudes toward mathematics, and attitudes toward the experimental procedures.

4. Significant gains ($p < .01$) occurred in computational skills and knowledge of mathematical concepts.

5. No significant differences in means were found between the experimental students and four seventh grade comparison groups with respect to knowledge of mathematical concepts.

6. The experimental group showed a significant gain in attitudes twoard mathematics ($p < .01$). Test anxiety decreased, but the decrease was not statistically significant.

7. The mean gain in attitudes toward mathematics of the experimental students was significantly higher ($p < .01$) than that of each of the four comparison groups.

8. Student reactions to all aspects of the classroom management procedure were favorable. Attitudes toward the procedure at the end of the year were more favorable than at the beginning of the year.

9. The classroom management procedure underwent modification in light of experience with its use. The changes were designed to increase the effectiveness of some procedures and to reduce the disadvantages of others.

Although the preparation of objectives and materials to accompany them was time consuming, the classroom use of behavioral objectives offered advantages to the students and to the teacher. The objectives guided the selection of appropriate conditions of learning, facilitated communication between the teacher and students of exactly what the students were to learn, and provided a basis for assessing and differentiating student achievement.

To improve this classroom technique, further work should be done in preparing objectives for use in teaching problem solving, discovery, generalization, and proof.

While the classroom management procedure was successful in providing for the individual needs of various types of students, modifications which produce greater achievement during the initial teaching phase of a topic should be investigated.

The success of the classroom management procedure with seventh grade students suggests that this technique should be investigated at other grade levels and in other mathematics courses.

Patterns of Verbal Communication in Mathematics Classes

James T. Fey, Teachers College, Columbia University

Systematic study of classroom verbal communication is a promising new direction in research on teaching. Knowledge of the characteristic classroom behaviors of teachers and students will provide basic concepts to be organized and interrelated by theory on instruction. Furthermore, observational instruments designed to describe patterns of verbal interaction will be a useful tool in experimental studies where teacher behavior is a central variable.

The structure of mathematics can be expected to exert a unique influence on patterns of verbal behavior in mathematics classes. Therefore this study undertook two main tasks: (1) To develop an instrument which describes the pedagogically and mathematically significant components of teacher-student interaction in verbal communication. (2) To use this instrument to describe patterns of verbal communication in five classes participating in the Secondary School Mathematics Curriculum Improvement Study.

Procedures

When each of five SSMCIS pilot classes began study of the chapter "Multiplication of Integers" in the experimental textbook, four consecutive meetings were (audio) tape-recorded. An observer, present at each recorded class meeting, made detailed notes, including description of all writing done at the chalkboard and special non-verbal action deemed necessary to complete the tape-recorded pictures of the class meeting. Each tape-recording was transcribed and coordinated with the observer's notes.

Next, using ideas from communication theory, logic and linguistics and concepts developed in earlier studies by Smith, Bellack, and Wright, a system was devised for describing the utterances in classroom discourse about mathematics.

Preliminary examination of the tapes showed that, following Bellack, classroom verbal communication could be considered to be a language game in which the players (teacher and students) make verbal moves toward the common goal, communication of mathematical ideas.

(1) The source of each move is the teacher or a student.

(2) The duration of a move is measured by the number of lines in the corresponding transcription of the recording.

(3) The pedagogical purpose of a move is either _structuring_ the discourse, _soliciting_ information or action, _responding_ to a soliciting move, or _reacting_ to a prior move of any type.

(4) The mathematical content of a move is the substantive topic under discussion.

(5) The mathematical activity to which a move contributes is either _developing_, examining, or _applying_ mathematical systems.

(6) The logical process of a move is the cognitive activity involved in dealing with mathematical content--_analytic_ (statements about the meaning and usage of language and symbolism), _factual_ (statements

that solicit or give information), evaluative (statements about the truth or appropriateness of a remark), or justifying (statements that support a previous remark by deduction, induction, opinion or authority).

When this system of analysis was fully developed and tested for reliability, each protocol was partitioned into a sequence of moves that were described according to source, duration, pedagogical purpose, content, mathematical activity, and logical processes. When the more than 8000 moves had been coded according to the above descriptive attributes, two analyses of the data were made. The first was a simple count of frequencies for the various possible types of moves; the second was an analysis of cycles in classroom discourse--sequences of moves with common mathematical content or activity.

Findings of Data Analysis

The data analysis produced a profile of verbal activity in the observed classes. Various patterns of source, duration, pedagogical purpose, content, mathematical activity, and logical process emerged (1) in individual class sessions, (2) in classes of particular teachers, and (3) in the twenty protocols taken as a whole. The basic play in the language game of observed classes was a three part verbal exchange beginning with a teacher move soliciting facts, followed by a student response stating a fact, and concluded by a teacher reaction evaluating this response. This pattern led to or was the result of the following specific patterns.

1. Each teacher spoke more moves and more lines (of tapescript) than all of his students. The ratio was 3 to 2 in terms of moves and 3 to 1 in terms of lines, the same ratios as discovered by Bellack in

his study of social studies classes in the senior high school.

2. Each teacher dominated the pedagogical functions of structuring (80 percent of these moves), soliciting (95 percent of these moves), and reacting (85 percent of these moves), leaving responding as the major student activity.

3. Over 50 percent of all moves were statements or questions of fact, 25 percent evaluations (mostly by teachers), and the remaining moves divided between justifying and analytic process.

4. Teachers not only dominated the pedagogical direction of classroom discourse, they controlled the content and mathematical activity by initiating over 90 percent of all content or activity cycles.

5. Content emphasis in all classes followed closely the sequence of the textbook chapter being studied, and the patterns of mathematical activity within content cycles also showed the influence of mathematical structure on the structure of discourse.

Findings 1, 2, and 3 confirm earlier results of Bellack in social studies classes, while finding five is in sharp contrast to the wide range of content emphasis in social studies classes. The concept of content and activity cycles was not used in comparable form by any earlier study.

Plans for Further Research

Teacher influence in shaping the direction of classroom activity differed from class to class, but the difference was primarily one of degree rather than kind. The roles of teachers and students in the classroom language game appear to obey certain implicit rules consistently from class to class. Nonetheless, the limitations of the present study with respect to grade level, number of classes observed, period of class time observed, and mathematical topic of discussion suggest many possibilities for further investigation.

The observational instrument must be tested in a wider range of classroom situations. When this is done, it can be used in studies of the relationship between teaching style and student achievement, of the impact of new curricula on classroom activity, and many other questions in which teacher and student behavior are central variables.

A Survey of High School Mathematics Teacher's Backgrounds and Their Opinions and Priorities with Respect to Current Issues in Mathematics Education

Max S. Bell, Assistant Professor
Graduate School of Education
The University of Chicago

Introduction

There have been within this century a number of recommendations advocating extensive changes in the materials and methods of school mathematics. But the actual changes in school practice in response to these recommendations have seldom been investigated systematically. Even when such investigations have been undertaken they have had little basis for comparison of supposed results with the prior situation, because there has been little usable data available from the period before the reform attempt. Furthermore, working teachers have seldom been consulted directly with respect to their opinions and priorities with respect to proposed reforms nor have their academic preparation and other experience been considered as aids to or barriers to instituting such reforms. As a possible corrective to this state of affairs, the survey reported in this paper aims to consult working mathematics teachers directly in order to do two things: (1) to provide information about teacher backgrounds and opinions that may be helpful in attempts to introduce certain new emphases in mathematics education and (2) to provide a status study which may be the basis for follow-up evaluations of the effect of these anticipated emphases a few years hence.

Those surveyed were 238 mathematics teachers, 100 of them in nine public high schools in the city of Chicago and 138 of them in ten public schools in the Chicago commuting area. These 19 schools are broadly representative of the geographic and socio-economic distribution in the Chicago metropolitan area and hence the survey included teachers working in a considerable variety of school situations. Detailed information was obtained from each teacher on his professional and educational background, courses taken in college, work experience other than school teaching, opinions on a number of current issues in mathematics education, and the priorities he would assign to a number of projects that have been proposed for changing mathematics education.

SURVEY PROCEDURES

The survey questionnaire was divided into three parts. The first part asks for detailed information on background, education, and non-teaching work experience; the second part asks for teacher opinions on certain issues in mathematics education; and the third asks the teacher to set priorities on certain projects that have been proposed for improvement of mathematics education. In addition, certain information was supplied from sources external to the questionnaire itself; for example, socio-economic information was taken from census figures, and information about colleges was taken from a Mathematical Association of America (MAA) publication. In both city and suburban schools the cooperation and rate of return was excellent. Furthermore, all but two of 239 participating teachers completed the questionnaires in every detail, surely an unusual result in studies of this sort.

The following statistical analyses were carried out:

Frequency counts, univariate statistics, correlation matrices, cross tabulations into contingency tables, factor analyses of the teacher opinion data, and regression analyses. Each part of the questionnaire and the results of the statistical analyses will be discussed.

SOME RESULTS:

1. The populations of city schools differ markedly from suburban schools with respect to such socio-economic status (SES) factors as median income and median years of education. While this does not necessarily say anything about the teachers in these schools, it no doubt affects the environment in which they work. With respect to the teachers in these schools, suburban school teachers are significantly more likely to be men; to have majored in mathematics for the Bachellor degree; to have Masters degrees, though often with majors in fields other than mathematics; to be more professionally active; to have more summer institutes; and to do more "moonlighting" for pay during the school year, usually as part-time coaches for extra pay.

2. Over half the teachers have been teaching for six years or less, and more than one-fourth for three years or less. Ninety-five percent teach mathematics only. About three-fourths of the teachers majored in mathematics for the Bachellor degree. About half of the teachers have

Masters degrees, and of these about three-fifths majored in mathematics or mathematics-education. They have an average of about twenty-four quarter courses (not hours) of college mathematics, with an average of about five of these identifiable as probably having a "modern" mathematics flavor. Seventy percent declare that they will definitely be teaching high school mathematics five years hence; six percent are not sure; and the rest say they will not.

3. Certain broad issues were built into the teacher opinion section of the questionnaire and these were confirmed by factor analyses. Several clusters of questions get at teacher opinions on the ease and importance of implementing in school practice a stress on the uses of mathematics and mathematical models; several get at teacher attitudes toward "reform" curricula and the desirability of introducing certain new content; and one cluster of questions gets at what teachers think should be done for those that are presently unsuccessful in learning mathematics. There is much to be said about the opinions thus expressed by teachers; a brief summary is that teachers are much more progressive in their attitudes than is sometimes supposed. Our dominant feeling based on these survey results is that we cannot conjure up teacher resistance as the main barrier to further reform in mathematics education, but must look elsewhere; perhaps to our willingness and ability to provide sufficient and perhaps massive support on a broad front to these teachers.

4. When asked to express agreement or disagreement with the importance of assigning a high priority to each of a number of projects proposed for mathematics education, teachers give substantial support to

nearly all of them. When asked to pick the five top priority projects on a national scale, these teachers select projects aimed at better understanding of how children learn and helping those that are unsuccessful under present practices; and projects aimed at improving elementary school mathematics education. They tend to assign low priority to further work on the college preparatory sequence of courses. When asked to pick five projects that they personally would like to work on they again select several that have to do with making mathematics more accessible to more youngsters and give low priority to more work on the college preparatory curriculum. They again reveal a quite progressive orientation, as well as good discrimination between what they see as important for someone to do on a national scale, and what they could personally make a contribution to, with the latter closely tried to projects that can and should have a close tie to actual classroom practice.

5. One of our main concerns was to explore the background and opinions of teachers that would bear on a possible increased emphasis on the uses of mathematics in school mathematics courses. We found that teachers feel that their college training in mathematics gave them little help in understanding such uses; that they have themselves had very few college courses in fields that apply mathematics; that they have had very little work experience themselves that has applied mathematics; that their attitudes are very positive with respect to the desirability of an increased emphasis on the uses of mathematics; and that they assign high priority to projects with that aim in mind. That is, they feel that more emphasis on the uses of mathematics is important, but that there is little

in their experience, training, or the materials they use that would assist them in effecting it.

6. Regression analyses revealed that in spite of a considerable amount of information on the backgrounds of teachers, it is not possible to account for very much of the variance in teacher opinions. That is, this survey was not very helpful in isolating the background and training factors that would "predict teacher attitudes, although there are a few suggestive results.

RESEARCH REPORT SESSION II

THURSDAY

12:30 p.m.--1:30 p.m. Jackson/Lincoln/Roosevelt (L)

Presider: Robert Bechtel, Purdue University, Calumet Campus, Hammond, Indiana

Bobbie M. Anthony, Chicago, Illinois

Relationships Between Classroom Average Arithmetic Achievement and Some Classroom Variables

John L. Creswell, University of Houston, Houston, Texas

A Problem of the Long-Range Effects of a Program of Curricular and Administrative Innovations on Achievement and Attitude of Disadvantaged Students

Ingrid B. Weise, Montgomery County Public Schools, Rockville, Maryland

Changes in the Motivational Determinants of Behavior in the Mathematical Classroom as the Result of a success Oriented Laboratory Experience

Modupe Taylor-Pearce, University of Alberta, Alberta, Canada

A Study of the Relative Effectiveness of Two Teaching Methods With Respect to (Divergent Thinking) Mathematical Creativity at the Grade XI Level, and the Construction of Tests for the Study

Relationships Between Classroom Average Arithmetic Achievement and Some Classroom Variables

Bobbie M. Anthony
Chicago, Illinois

Many attempts have been made to discover factors underlying arithmetic achievement. The effects of many particular classroom characteristics on such achievement have been examined to, hopefully, obtain evidence on how arithmetic achievement could be fostered. Recently, the writer concluded a study during which analyses were made of relationships between classroom average arithmetic achievement and both single classroom environmental variables and combinations of these variables. The purpose of this article is to report on these relationships. Practical considerations prohibit inclusion of a detailed description of the study. However, the details are fully discussed elsewhere(1).

Twenty-one fifth-grade public-school classrooms were observed six times each. They were part of a school system servicing a Midwestern community of approximately 60,000 and were obtained by randomly selecting the ten schools housing them from strata based on standardized achievement scores. Classroom observations were supplemented by interviewing the classroom teachers.

Data obtained through observations and interviews were rated on fifty-one scales designed to measure fifty-one characteristics associated with classroom environmental variables. The basis for selection of the variables and characteristics was a procedure suggested by Bloom (2). A principal component analysis of the ratings yielded two reliable factors, reliability being determined by a variant of Hoyt's method (3). These factors or components were termed Stimulation and Individual-emphasis dimensions--or process variables--of the classroom environment because of the particular groups of characteristics with significant loadings on the factors. In this article, these factors or dimensions will be called Factor 1 and Factor 2, respectively.

Factor measurements were in terms of factor scores and, also, sums of ratings on groups of characteristics loaded on the factors. Results using both types of measurements in subsequent similar analyses were similar, though not identical in all cases. In a few analyses, only one type measurement was used. Results of similar analyses will be reported here for only the ratings-sum type of measurement. In accordance with results of regression and correlation analyses, the scores on the two classroom dimensions were combined. The resulting sum was considered a classroom educational environment index and will be referred to as E.E.I. in this article. Although ratings on all fifty-one characteristics were used to calculate E.E.I., only characteristics used to calculate or construct shortened versions of E.E.I. will be described here later. The shortened versions will be referred to as 14var and 13var, which is identical to 14var minus a characteristic designated Var7.

Initial arithmetic scores were based on the Elementary Battery of the Metropolitan Achievement Tests. Terminal scores were based on the Intermediate II Battery of The Stanford Achievement TEsts. However, a table of equivalents, made available by the firm which publishes both tests, was used to make the scores comparable. Class arithmetic averages as well as class IQs were medians, rather than means.

Statistics for regression analyses made to obtain information on the predictive validity of various versions of the E.E.I. are displayed in Tables 1 and 2 for classroom average arithmetic computation and arithmetic concept-and-problem-solving achievement, respectively. Simple correlations between arithmetic averages and other classroom variables are shown in Table 3. Where necessary, a general description of variables listed in Table 3 is given in Table 4.

The results shown in Tables 1 and 2 indicate that the E.E.I.; both environmental dimensions (factors); a shortened version of the E.E.I.; and the single characteristic--Var7--were all significantly related to terminal arithmetic achievement when initial achievement was controlled. Some of the relationships were stronger for arithmetic concepts and problem solving than for arithmetic computation. In particular, the E.E.I. was significantly related to terminal concept-and-problem-solving achievement when both initial achievement and IQ averages were controlled. When teacher experience in years, the E.E.I., initial achievement and IQ averages were all used as predictors, no one of these variables had a significant partial correlation when the others were controlled. However, the E.E.I. and initial achievement seem more related to terminal achievement than the other two variables. The regression analyses suggest that some classroom environmental variables to influence fifth-grade arithmetic achievement and that it is possible to predict this achievement from measurements of these variables.

REFERENCES

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2. Bloom, Benjamin S. *Stability and Change in Human Characteristics*. New York: John Wiley and Sons, Inc. 1964.

3. Hoyt, Cyril. "Test Reliability Estimated by Analysis of Variance," *Psychometrika*, VI (1941), 153-60.

TABLE 1
SOME REGRESSION STATISTICS FOR RELATIONSHIPS BETWEEN CLASSROOM AVERAGE ARITHMETIC COMPUTATION ACHIEVEMENT AND SOME OTHER CLASSROOM VARIABLES[a]

Independent Variables[b]	Multiple Correlation, R	R^2	F-Ratio for Significance of Correlation	
			Partial	R
E.E.I.			5.71[c]	
A.C.1			3.01	
	0.704	0.495		8.83[d]
E.E.I.			3.75	
A.C.1			2.84	
Median IQ			0.02	
	0.704	0.495		5.57[d]
A.C.1			7.43[c]	
Var7			5.59[c]	
	0.702	0.493		8.74[d]
13var			4.74[c]	
A.C.1			3.27	
	0.688	0.473		8.09[d]
A.C.1			2.96	
E.E.I.			2.83	
Teacher experience (years)			0.43	
Median IQ			0.01	
	0.713	0.509		4.14[d]

[a] The dependent variable in each of the five analyses was A.C.2--classroom terminal average arithmetic computation achievement.

[b] E.E.I. is a classroom educational environment index; A.C.1 is the initial counterpart of A.C.2.

[c] Significant at the 0.05 level

[d] Significant at the 0.01 level

TABLE 2
SOME REGRESSION STATISTICS FOR RELATIONSHIPS BETWEEN CLASSROOM AVERAGE ARITHMETIC CONCEPT-AND-PROBLEM-SOLVING ACHIEVEMENT AND SOME OTHER CLASSROOM VARIABLES[a]

Independent Variables[b]	Multiple Correlation, R	R^2	F-Ratio for Significance of Correlation	
			Partial	R
E.E.I.			13.92^c	
APP.C.1			0.66	
	0.734	0.539		10.53^c
E.E.I.			8.07^d	
Median IQ			0.63	
APP.C.1			0.42	
	0.745	0.556		7.09^c
Var7			7.15^d	
APP.C.1			3.34	
	0.644	0.415		6.38^c
13var			6.82^d	
App.C.1			1.14	
	0.638	0.407		6.18^c
Factor 1			13.94^c	
APP.C.1			0.33	
	0.734	0.539		10.54^c
Factor 2			10.28^c	
APP.C.1			1.76	
	0.693	0.480		8.30^c

[a] The dependent variable in each of the six analyses was APP.C.2--classroom terminal average arithmetic concept-and-problem-solving ahcievement.

[b] E.E.I. is a classroom educational environment index; APP.C.1 is the initial counterpart of APP.C.2; Table 4 explains the other variables.

[c] Significant at the 0.01 level

[d] Significant at the 0.05 level

TABLE 3

COEFFICIENTS OF CORRELATION FOR CLASSROOM AVERAGE ARITHMETIC ACHIEVEMENT AND OTHER CLASSROOM VARIABLES[a]

Other Classroom Variables[b]	Arithmetic Computation		Arithmetic Problem Solving and Concepts	
	Initial	Terminal	Initial	Terminal
A.C.2	0.579			
APP.C.1	0.761	0.617		
APP.C.2	0.354	0.619	0.427	
Average days absent	0.158	0.393	0.174	0.228
Teacher experience (years)	0.055	0.263	-0.055	0.200
Median IQ	0.306	0.389	0.381	0.535
Factor 1	0.546	0.613	0.476	0.729
Factor 2	0.428	0.627	0.327	0.655
E.E.I.	0.514	0.641	0.428	0.723
13var	0.508	0.615	0.413	0.608
Var7	0.256	0.532	0.187	0.554
Var19	0.274	0.353	0.261	0.484
Var20	0.595	0.440	0.640	0.492
Var31	0.258	0.387	0.215	0.384
Var2	0.356	0.455	0.310	0.356
Var27	0.085	0.344	0.069	0.431
Var41	0.275	0.589	0.272	0.505
Var43	0.341	0.620	0.266	0.469
Var44	0.465	0.538	0.321	0.372
Var45	0.491	0.361	0.184	0.230
Var47	0.350	0.320	0.232	0.469
Var48	0.267	0.269	0.308	0.517
Var49	0.461	0.500	0.420	0.548
Var50	0.592	0.472	0.269	0.185
14var	0.485	0.616	0.391	0.613
Teacher oral communication	0.251	0.494	0.161	0.559
No. of calls for order	-0.204	0.093	-0.080	0.394
14var + teacher oral communication	0.459	0.667	0.350	0.696
Class size	0.123	0.242	0.095	0.207
Class percent of males	-0.312	-0.045	-0.067	-0.179

[a] N = 21

[b] Where necessary, variables will be described in Table 4; some abbreviations are explained in the text and in footnotes beneath Tables 1 and 2.

TABLE 4
DESCRIPTION OF SOME VARIABLES ABBREVIATED IN COLUMN ONE OF TABLE THREE[a]

Variable	Variable Description
Var2	Number of teaching devices used at least twice
Var7	Total variety in supplies, apparatus, equipment and materials used by pupils
Var19	Number of distinct kinds of supplies, apparatus, equipment and materials used by pupils
Var20	Number of kinds of tests given
Var27	Frequency of intensity in teaching devices (3-dimensional, illuminated, appealing to several senses)
Var31	Frequency of pupils working independently
Var41	Distinct types of teaching devices or equipment (not text or chalkboard) (observed)
Var43	Number of instances of teacher negative support (rejecting, scolding, ignoring, blaming, etc.) (observed)
Var44	Ratio of teacher positive affect to total teacher affect
Var45	Number of instances of observed achievement rewards
Var47	Number of academic 3-dimension displays and exhibits observed
Var48	Number of academic displays and exhibits observed.
Var49	Number of instances of novelty or real-live representation in procedures; activities; room decorations; pupil materials, supplies, apparatus, equipment; teaching aids (observed)
Var50	Number of instances of teacher positive support (accepting, encouraging, praising, speaking markedly pleasantly, etc.) (observed)
13var	A combination of all above variables except Var7
14var	13var + Var7
Teacher oral Communication	Amount of teacher speech of certain types presumed to be achievement inducing

[a] Factor 1 and Factor 2 were discussed at length above.

A Problem of the Long-Range Effects of a Program of Curricular and Administrative Innovations On Achievement and Attitude of Disadvantaged Students

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I. PURPOSE AND SIGNIFICANCE

The implementation of the Civil Rights Act of 1964 has brought teachers in the Houston area into contact with students of diverse ethnic and social backgrounds for the first time. Attention should therefore be given to necessary administrative and curricular modivications growing out of the diversity of the student population and the inexperience of the faculty in these situations.

Accordingly, a twenty-day highly concentrated, tailored school program involving curricular and administrative innovations was held in the summer of 1967. Analysis of data collected from this program indicated that each student gained an average of four months in mathematics achievement.

There was immediate need to determine if this achievement had any carry-over effect in the actual classroom.

II. CONCEPTED FRAMEWORK

The study explored some long-range aspects of curricular and administrative innovations as they related to the academic achievement and attitude of students from impoverished homes. The primary objectives included the following:

a. determining to what extent curricular and administrative innovations have enabled disadvantaged youngsters to develop and maintain:

(1) more positive feelings toward the learning processes;

(2) more positive attitudes toward formal schooling as determined by attendance; and

(3) proficiences in mathematics as determined by grade point average;

b. determining the relationship between academic achievement and attitudes of disadvantaged children as measured by questionnaires, observations and visitations;

c. determining the extent of growth or regression in mathematics as measured by achievement tests.

III. PROCEDURES

A. General Design

Phases I and II of the program was conducted in the Summer of 1967 for twenty days duration. In Phase I, 200 4th, 5th, and 6th grade students from disadvantaged homes were administered pre-tests in mathematics and attitude. Then each child was given a twenty-day program of mathematics, *language arts, and human relations training (one hour in each area each day) using team teaching, concrete materials, visuals, programmed instruction, etc. Each group of ten studetns was taught by a team of three teachers.

At the end of the twenty-day period post-tests were administered (Phase II). Results of the analysis of the data were encouraging. The

*only mathematics and attitude are considered in this paper.

W.S.O.E. funded the follow-up study (Phase III) through a small Research Grant.

In Phase III, 127 of the original 200 students were tested for attitude and arithmetic achievement in early June, 1968. Due to incomplete records, only eighty of the 127 could be used in the follow-up.

In august 1968, a random sample of thirty of the eighty students in the follow-up were selected. The homes of these students were visited, from which an interview form was completed by the interviewer.

B. Data Collection

Four types of data were collected: (1) data from school records, i.e., grade point average, attendance, and mental ability scores; (2) scores from a semantic distance scale designed to assess attitude toward school processes; (3) scores from standardized achievement tests in mathematics and language arts (word knowledge and reading comprehension); and (4) data collected from home interviews, i.e., a.) parental attitudes; b.) parental perception of child's attitudes and c.) home environment.

C. Data Analysis

The data was analyzed through a multiple classification analysis of variance, and processed on the SIGMA 7 BCM Computer through the facilities of the University of Houston Computer Center.

D. Findings

1. attendance - no significant difference in school attendance was found between 1966-67 and 1967-68.
2. mathematics grade point average - a significant difference between means was found beyond the .06 level of confidence.
3. mathematics achievement - a significant difference beyond the

.01 level of confidence was found between the means. Critical ratios indicated Phase I mean less than Phase III mean. (Progression)

4. attitude - analysis of variance indicated that no significant differences existed among the means.
5. home visitation - data from the home visitation interviews revealed that no adult male lived in 19 of the 30 homes visited and 10 of the 19 mothers were unemployed. Twenty-three of the mothers perceived their child as liking school prior to the summer program, but experiencing a more positive attitude following the program. Twenty-two reported the child to have frequently mentioned the program during the nine months following. Parental ambitions for the child were usually not expressed, although when the parent did express an ambition, it was for a profession (six named nursing; others were lawyers, priest, and architect, for a total of 12 responses.)

   The parents rated the summer program staff as excellent while rating the public school staff as good.

   The thirty parents unanimously agreed that the summer program was beneficial to the children, and mentioned improvement in school work and social relations as an effect of the program.

IV. CONCLUSIONS

An increase in grade point average in mathematics was found to be significant beyond the .06 level. This seems to indicate a decided carry-over effect, particularly since the original study showed a gain in mathematics achievement significant at the .01 level.

Analysis of variance showed highly significant differences between the means on the California Achievement Test in Arithmetic. These differences were in a positive direction, and reflected the gains in arithmetic achievement of the original study. It would seem, therefore, that for this population, the summer program had a decided long range effect on the mathematics achievement. It is surmised that much of this may indeed be due to the innovations in mathematics used in the summer program, which resulted in a positive change of attitude toward mathematics on the part of the students.

Item analysis of the Semantic Distance Scale indicated the least favored items to be in the areas of teacher and classroom, with several of these items related to fairness of the teacher. This could possibly be interpreted as the tendency of lower-class children to resent the middle-class values of the teacher. Studies have shown that the social standing of the child has more to do with the grade assigned by the teacher than the measured achievement of the child.

Changes in the Motivational Determinants of Behavior in the Mathematics Classroom as the Result of a Success Oriented Laboratory Experience

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Introduction

The word motivation is often used in explaining failure in the classroom. When a bright and otherwise well adjusted youngster fails miserably in math, or spelling, or geography, the assumption is often made that he is not motivated to achieve in this or that subject area. When such a youngster is identified, efforts are made to raise motivational levels with regard to one or more academic areas, and more often than not, such an attempt is no more or less than a stab at making the subject or subjects more interesting for the pupil. The classic test of whether or not a change in motivational posture has occurred is to determine whether or not a change in achievement level takes place subsequent to changes of curriculum and/or procedures which were hypothesized to raise motivation on the part of the individual in question.

An alternative to the classical approach would be to look directly for changes in "pupil motivation" which follow curricular and/or procedural changes in the classroom; that is to say for personality changes rather than grade changes.

Theoretical Model

The widely used term "motivation" is an inclusive concept which can be broken down into sub-concepts. For the purposes of this investigation, the theoretical model of Tolman[1], as expanded by Atkinson[2], is utilized. This theoretical model postulates that strength of motivation in any particular situation is dependent upon a multiplicative function of the variables--motive, expectance, and incentive.

1. Motive

   "A motive is conceived as a disposition to strive for a certain kind of satisfaction . . . such as achievement, affiliation, power . . ."[2]

2. Expectancy

   "An expectancy is a cognitive anticipation, usually aroused by cues in a situation, that performance of some act will be followed by a particular consequence, given the act."

3. Incentive

   Incentive " . . . represents the relative attractiveness . . . or the relative unattractiveness of an event that might occur as a consquence of some act." (For example, where getting money is the goal; varying the amount would be one way to vary the incentive.)

Rationale of the Program

Among the rationale of the program previously described is the underachiever in mathematics--" . . . the student who has experienced repeated failure in his mathematics clases to the extent of having become a reluctant learner." The strong implication is that if a student can be made to experience success, that is to say his expectancy can be made to change, he will become "motivated" and will achieve on a higher level.

Procedure

The Instrument

A risk-taking design was employed as follows: Four sets of five questions each were constructed in each of four content areas (general information, vocabulary, history, and mathematics.) Each of the five questions in each set were of varying levels of difficulty; each set contained one question labeled as follows: very easy, easy, medium, hard, and very hard. Each question was typed on a separate slip of paper with the designation of difficulty on the reverse side.

The Testing Stiuation

On two occasions, the first day and the last day of the program, the above instrument was applied. On each occasion it was explained to the students that they were going to play a game as follows. Questions were going to be presented. Each time their turn came, they could pick one question out of five (labeled as described above.) Each question carried a different point value as follows: Very Easy, one point; Easy, two points Medium, three points; Hard, four points; and Very Hard, five points. The point system was thoroughly explained. It was stated that the questions would cover four subject areas (as described above) and that the subject area would be announced. The object was simply to acculULate as many points as possible. The testing proceeded as follows. A subject area was announced. Each student came to a table at the front of the room and selected a question from one of five piles--plainly marked as to level of difficulty and point rating. (The questions themselves could not be seen as they were face down on the table.) The entire class cycled twice through the sets of questions in four areas.

Rationale of the Study

Simply stated, the rationale is this; if a student has learned to expect failure *vis-a-vis* mathematics, that is to say he has an *expectancy* of failure, he will minimize risks whenever possible. He will choose questions which are described in a way consonant with his *expectancy*. If over the course of any series of events the student comes to learn that he can achieve success, that is to say he developes a positive *expectancy*, when he chooses questions described in a way consonant with his level of *expectancy*, they will be questions described as more difficult than previously chosen questions.

This is the hypothesis tested: the group will, at the end of the program, show an increase in level of difficulty of question chosen in the academic area of mathematics. No rise in level of difficulty in other academic areas was expected.

Results

Nineteen boys and girls were present for both testing sessions. Scores are computed on the basis of the point rating system of difficulty (described above) of questions chosen without regard to whether or not the questions were correctly or incorrectly answered. The scores for the nineteen individuals are shown in Table I below. Pre and post test means were compared to a T test as shown in Table II.

Table I

Raw Scores of Students on Pre- and Post-Test Runs

	PRE-TEST								POST-TEST								
Student	Information		Vocabulary		History		Arithmetic		Information		Vocabulary		History		Arithmetic		Student
	Run		Run		Run		Run		Run		Run		Run		Run		
	#1	#2	#1	#2	#1	#2	#1	#2	#3	#4	#3	#4	#3	#4	#3	#4	
1	3	4	3	4	4	4	3	4	4	5	4	5	4	4	4	5	1
2	3	2	3	2	3	2	3	4	3	3	3	2	3	2	4	5	2
3	3	3	3	3	4	4	3	4	4	4	3	4	4	5	4	4	3
4	4	3	3	3	3	3	3	3	3	3	3	3	4	3	4	4	4
5	3	4	5	3	3	4	2	3	4	4	5	4	5	5	3	3	5
6	3	4	2	3	2	3	3	4	3	3	2	3	2	3	3	2	6
7	3	3	2	3	2	3	2	4	3	3	3	3	3	4	4	5	7
8	5	4	3	4	4	4	2	3	4	3	4	4	4	3	2	3	8
9	3	4	2	4	2	3	2	4	3	3	3	4	3	3	4	3	9
10	4	5	3	4	4	3	3	3	3	4	3	4	3	3	4	4	10
11	3	2	4	3	4	4	3	3	4	4	3	3	2	2	4	4	11
12	5	4	4	1	5	1	3	5	1	4	2	4	3	3	5	5	12
13	3	4	2	3	2	3	3	4	3	4	3	3	3	3	3	5	13
14	5	3	3	3	4	3	4	5	4	5	3	3	3	4	4	5	14
15	3	4	3	3	2	4	4	5	3	3	3	3	3	4	4	5	15
16	4	4	2	4	3	5	2	5	4	4	2	2	5	5	4	5	16
17	2	3	2	3	3	3	3	4	4	3	3	3	3	3	3	4	17
18	3	3	3	4	4	3	4	3	3	3	3	3	2	3	2	3	18
19	4	5	4	3	5	4	3	2	4	3	4	4	4	3	4	4	19

Table II

Pre- and Post-Test Means Compared

	Mean Pre-test Run #1	Mean Post-test Run #3	Diff.	Level of Significance	Mean Pre-test Run #2	Mean Post-test Run #4	Diff.	Level of Significance
rmation	3.473	3.368	0.105	---	3.578	3.526	0.050	---
bulary	3.105	2.947	.158	---	3.368	3.157	.211	---
ory	3.315	3.315	.000	---	3.421	3.315	.106	---
hmetic	2.894	3.631	.737	$P < .01$	3.789	4.210	.421	$P < .20$

The mean post-test scores for arithmetic (3.631 and 4.210) are significantly higher than the pre-test means to which they were compared (2.894 and 3.789).

Interpretation

During the course of the special summer school program, the participants as a group developed a willingness to "risk" choosing questions which were described as being of a higher level of difficulty over the ones they chose at the beginning of the program. This may indicate a change in expectancy of success vs. failure vis-a-vis mathematics, and thus, a change in motivational posture toward that subject.

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[1] Tolman, E. C. Principles of Performance. *Psychological Review*, 1955, 62, 315-326.

[2] Atkinson, J. W. *Motives in Fantasy, Action, and Society*. Van Nostrand, New York, 1958, pp. 322-324.

A Study of the Relative Effectiveness of Two Teaching Methods With Respect to (Divergent Thinking) Mathematical Creativity at the Grade XI Level, and the Construction of Tests for the Study.

Modupe Taylor-Pearce
University of Alberta

PURPOSE AND SIGNIFICANCE OF RESEARCH

Arguments have been advanced for and against the possibility of encouraging creativity in students through certain methods of teaching. While, for example, Hohn[1] and Davis[2] suggest correlation between discovery teaching and encouraging creativity, Ausubel[3] finds it totally unrealistic to suppose that even the most ingenious techniques could stimulate creative accomplishment in children of average endowment.

The purpose of the experiment is to investigate two methods - an expository type and a discovery type - in terms of their relative effectiveness in encouraging (divergent thinking) mathematical creativity.

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[1] F. E. Hohn, "Teaching Creativity in Mathematics," The Arithmetic Teacher, March, 1961.

[2] Robert B. Davis, "Discovery in the Teaching of Mathematics," In Lee S. Shulman and Evan R. Keislar, (Eds.), Learning by Discovery. A Critical Appraisal, (Chicago: Rand McNally and Company, 1966).

[3] David P. Ausubel, "Some psychological and educational limitations of learning by discovery," The Arithmetic Teacher, May, 1964, p. 301.

Any method that could be shown to consistently result in encouraging mathematical creativity in students would be of considerable interest to those who consider creativity in mathematics as of pressing importance. There has been considerable recent interest in creativity, and many think that encouraging creativity is vital for a changing world with unusual challenges.

The setting of this study is in the senior high school level, and little appears to have been studied at this level in the discovery-expository investigations. This investigation should provide some information at this level.

The study should also provide some further information in a specific aspect of a field in which, although much research has been conducted, few general trends have been found.

THE DESIGN

The study is based on 231 students from ten classes taught by five Grade XI teachers in the Edmonton Public School System. Each teacher taught two methods, using the expository-type method in one class, and the discovery-type method in the other. The teachers were specially trained at a methods in-service training course, to teach both methods. The methods were defined in detail and the teachers taught special units of linear and quadratic equations on materials prepared at the University. The investigator had to deal with intact classes, but from all evidence available, the classes were not originally selected on any basis that affects the criterion variables. Accordingly, the students were considered as random samples of some population. The order of teaching was random.

The students were given a pre-test and a post-test. Both tests were prepared by the investigator, and administered by the respective teachers. Forty minutes were allowed for each test.

THE TESTS

The investigator based his tests on the findings of Guilford that most of the more obvious contributions to creative thinking are in the divergent thinking production category and that the factors of fluency, flexibility and originality are in that category.[4] The investigator was also influenced by the research of Evans[5] and Prouse[6] in construction the tests. The Pre-test was based on the mathematics that the students had done prior to embarking on the experiment, and the Post-test was based entirely on what was taught during the experiment. Each question was designed to evaluate fluency, flexibility and originality. The tests had all been evaluated by a reference group of university professors and graduate students, and the final form was based on the items that had received maximum approval. Pilot studies were carried out on the tests, and estimates of reliabilities were obtained using analysis of variance techniques as may be found in Winer,[7] pages 124-132.

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[4] J.P. Guilford, "Three Faces of Intellect," American Psychologist, 1959, pp. 469-479.

[5] E.E. Evans, Measuring the Ability of Students to Respond in Creative Mathematical Situations at the Late Elementary and Junior High School Level, (unpublished Ph.D. dissertation, University of Michigan, 1964).

[6] Howard L. Prouse, "Creativity in School Mathematics," The Mathematics Teacher, December, 1967.

[7] B. J. Winer, Statistical Principles in Experimental Design, (McGraw-Hill, 1962).

SCORES

Each subject received a fluency score, a flexibility score and an originality score. One fluency mark was awarded for each appropriate response. One flexibility mark was awarded for each distinct flexibility class to which a student's set of responses belonged. A flexibility class was considered to be a set of fluent responses having an underlying generating principle. The originality scores for the distinct flexibility classes to which his fluent responses belonged. The originality score for a flexibility class was awarded as an index of the degree of uncommonness of the flexibility class. The uncommonness was determined by the proportion of the number of subjects whose responses belonged to the class, to the total number of subjects.

PRESENT STATE OF ANALYSIS

The investigator expects to complete the analysis within the next month.

STATISTICAL PROCEDURES AND HYPOTHESES

Analysis will be conducted with respect to each of four criteria - fluency, flexibility, originality, and total.

A single factor analysis of variance will be conducted on the pre-test scores for each criterion on hypothesis 1.1, that the discovery-type group and the expository-type group are homogeneous with respect to the criterion, prior to the experiment.

If hypothesis 1.1 is substantiated than decisions will be made on the basis of the post-test scores. A two factor analysis of variance model will be used, with methods and teachers being the factors. Hypothesis 2.1 will be tested that on the basis of the post-test criterion scores, there is no interaction between methods and teachers.

If hypothesis 2.1 is substantiated, then hypothesis 3.1 will be tested that there is no difference in the effects of the two methods for the two groups. Also hypothesis 4.1 will be tested that there are no differences in the effects of the teachers each considered over the methods.

If hypothesis 2.1 is not substantiated, then the simple main effects of the methods for each teacher will be tested and the simple main effects of the teachers for each method will be tested.

If hypothesis 1.1 is not substantiated, then a two factor analysis of covariance will be conducted, the factors being as above, and the dependent variable being the adjusted post-test scores, using the pretest scores as covariate.

The two factors will be regarded as fixed.

CONCLUSIONS

Conclusions will be drawn on the basis of the results of the analysis. The investigator, after marking the tests feels that some of the responses were noteworthy. A sample of questions and responses is attached as an appendix to this paper.

APPENDIX

SAMPLE OF QUESTIONS AND RESPONSES

QUESTION: Think out true statements that make use of the idea of a kasep in the sense defined below. Write down ten of them.

Definition: A kasep is an integer divisible by 39.

1. Kaseps are closed with respect to addition, subtraction, multiplication, but not with respect to division.
2. The largest number of kaseps between A and B, where $B - A = 100$, is 3, and the smallest is 2.
3. The greatest negative kasep is -39, and the smallest positive kasep is 39.
4. Kaseps are not primes, and are divisible by 1, 3, 13, 39.
5. Kaseps may be expressed in set notation as the relation $K = k$: $k = 39n$, $n \varepsilon I$ The graph of $K = 39n$, is linear.
6. Every integer can be expressed as the quotient of a kasep and 39.
7. There is no multiplicative identity in the set of kaseps. There are no multiplicative inverses either.
8. If we designate a kasep as $\text{kasep}(n) = 39n$, then $\text{kasep}(2) + \text{kasep}(3) = \text{kasep}(5)$.

QUESTION: The following three numbers are arranged according to a definite pattern. Try to think out five possible values of x and in each case explain briefly how you obtained this value. 25, 625, x.

1. $x = 5^6$ nth term is 5^{2n}.
2. $x = 5^{(8)}$ nth term is 5^{2^n}.
3. $x = 1225$ nth term is $600n - 575$.
4. $x = 5^5$ nth term is $5^{(6-2^{(3-n)})}$.
5. $x = 10{,}000$ nth term is $\frac{3n - 1}{8} \times 10^{n + 1}$
6. $x = 45^2$ nth term is $[25 + (n-2)\ 20]^2$
7. $x = 1825$ $T_{n+1} = 10$ [Sum of digits of $T_n]^2 + 135$.
8. $x = 625a + b$ Sequence is $25,\ 25a + b = 625,\ 625a + b$.

QUESTION: Write down three sets of integers (m, n, q) qhich satisfy the equation: $m^2 + n^2 = q^2$. The set (3, 4, 5) is one such set. Write down seven sets of integers (m, n, q) which satisfy the equation $m^3 + n^3 = q^3$.

$m^2 + n^2 = q^2$	$m^3 + n^3 = q^3$
1. $(\pm 3a, \pm 4a, \pm 5a)$	1. $(0, 0, 0)$
2. $(\pm 5a, \pm 12a, \pm 13a)$	2. $(a, -a, 0)$
3. $(\pm 7a, \pm 24a, \pm 25a)$	3. $(0, a, a)$
4. $(\pm 9a, \pm 40a, \pm 41a)$	
5. $(\pm 8a, \pm 15a, \pm 17a)$	
6. $(0, 0, 0)$	
7. $(a, 0, a)$	

QUESTION: On the piece of graph paper provided, mark out two points A(2, 4) and B(-2, 4).

(a) Write down any three relations whose graphs contain these points.

(b) Draw seven different figures which pass through these points.

(a) x, y, : $y = 4$

(b) x, y, : $y = x^2$

(c) x, y, : $y = 2x^2 - 4$

(d) x, y, : $x^2 + y^2 = 20$

(e) x, y, : $y^2 = 2 \mid x^3 \mid$

(f) x, y, : $y + |x| = 6$

(g) x, y, : $-2 \leq x \leq 2$,.

QUESTION: The following three functions are arranged in a definite pattern. Try to think out five possible functions that could stand in place of f(x), and in each case explain briefly how you obtained the function:

$$(x^2 + 2x + 1), (x^2 + 6x + 9), f(x), \ldots.$$

1. $F_n = \{x + 3^{1/2(3^n-1)}\}^2$
2. $F_n = [x + (2n - 1)]^2$
3. $F_n = (x + 3n-1)^2$
4. $F_n = [x + (8n - 7)^{1/2}]^2$
5. $F_n = x^2 + (4n - 2)x + 8n - 7)$
6. $F_n = x^2 + 2(2n -1)x + \frac{(2n - 1)^2}{(2n - 3)}$

RESEARCH REPORT SESSION III

FRIDAY

10:30 a.m.--11:30 a.m. Jackson/Lincoln/Roosevelt (L)

Presider: Edgar Howell, State University of New York at Oswego, Oswego, New York

Nicholas Vigilante, University of Florida, Gainesville, Florida

How Children Perceive Relations

L. Ray Carry, University of Texas, Austin, Texas

Interaction of Visualization and General Reasoning Abilities With Curriculum Treatment in Algebra

Robert E. Rector, Indiana State University, Terre Haute, Indiana

The Relative Effectiveness of Four Strategies for Teaching Mathematical Concepts

William A. Miller, Central Michigan University, Mt. Pleasant, Michigan

The Acceptance and Recognition of Six Logical Inference Patterns by Secondary Students

How Children Perceive Relations

Nicholas Vigilante
University of Florida

This study is an attempt to gain some insight as to how children perceive relationships. Since the emphasis in modern mathematics is now in terms of recognizing patterns, it becomes a very interesting question as to what kind of relationships children are albe to perceive and at what age they are able to understand the more sophisticated sets of ordered pairs. Relations can be expressed in terms of set notations and from this abstract point of view, a relation is a set of ordered pairs. For instance, if one considers (a,b) as representing an ordered pair of things, in which _a_ is understood to precede _b_, the ordered pair may or may not belong to the relation R. To illustrate: consider all the ordered pairs of positive whole numbers such as (5,1), (6,4), (3,2), in which some number is subtracted from the first to give the second. In other words, consider all the ordered pairs of the form (a,b) where a-k = b, and k is a positive whole number. The relation defined by this set of ordered pairs (R) is the relation "greater than". Among the other important characteristics of relations are some of the various properties defined below:

Transitive: if aRb and bRe, then aRe

Intransitive: if aRb and bRc, then aR'c

Symmetric: if aRb, then bR'a

Asymmetric: if aRb, then bR'a

Reflective: for all a in A, aRa

Irreflexive: for all a in A, aR'a

Universal: for all a, b in A, aRb

Some types of relations which have certain combinations of these properties are those which are (1) both transitive and symmetrical, and (2) those which are both transitive and asymmetrical. Usually these two types of relations are the most significant in mathematics.

A question of considerable importance in developmental psychology concerns the specification of the age at which children comprehend the implications of these more complex relationships. According to Piaget, the idea of an independent measuring tool is the most complex of the stages a child will go through when asked to build something of the same height as a model. Piaget describes a task where a child is asked to build a tower out of blocks the same height as a model. The tower the child builds is on a table either higher or lower than that of the model and it is only at the final or most complex stage that the child is able to synthesize a division into transitive parts and substitute a measuring device to make his tower the same size as the model.

Bruner and Oliver speak of associative grouping as what people do when they relate one thing to another. Whether they are asked to group two or more words, events, or objects which occur in succession or simultaneously, Bruner and Oliver take the point of view that associations do not just happen but are governed by rules and are the result of complex transformations made by the organisms. In the article to which I refer, they concerned themselves with the development of equivalence transformations in children.

There is ample proof throughout the literature that the cognitive processes children go through in order to see and perceive relationships accurately are ordered and complex and any information that can be gathered to shed light on these processes will help in the understanding of learning. The present study is an effort to find out whether children of various ages recognize complex relationships in familiar contexts and at what maturation level the various relations can be understood.

Specifically, a set of sixteen items illustrating symmetric, asymmatric, transitive-symmetric, and transitive-asymmetric relations was administered to a sample of second- and fourth-grade students and the resulting data were analyzed with respect to the interaction of age with types of logical relationships.

METHOD

Subjects

A total of 52 subjects participated in the study. Twenty-four were fourth graders and 28 were second graders. The sample included 12 boys and 12 girls from the fourth grade, 14 boys and 14 girls from the second grade. The subjects were all enrolled at the P.K. Yonge Laboratory School.

Test Materials

The instrument used was a sixteen-item questionnaire. The questions consisted of four items representing each of the following four relations: symmetric, asymmetric, transitive-symmetric, and transitive-asymmetric. The phrasing of the items closely followed the definitions of the properties, naming the relation and using arbitrary names as elements. The questions could be answered by yes or no, yes indicating they felt the relationship existed and no indicating they did not believe the relationship to be true. All questions included the names of only one sex. An attempt was made to phrase these relations in terms of contents familiar to the age group of the subjects. Each question was given a number and then positioned on the tesxt according to a table of random numbers.

Procedure

In order to enlist the subjects' cooperation, they were instructed that "I would like to play a kind of detective game with you, etc., where you will be given clues to solve problems. Your job is to decide whether the clues tell you to answer the question 'No or 'Yes,' etc." Each question was then read aloud. Approximately 20 seconds were allowed for answering each of the questions. No questions were answered once the test was begun. The questionnaire was administered in the regular classrooms with the teachers present.

RESULTS AND DISCUSSION

Usable data were collected from all 28 second graders and 24 of the 25 fourth graders. The data of one fourth grader were discarded since independend evidence indicated that the individual generally functioned below grade level. A t-test comparing the mean numbers of items correct indicated that the fourth graders displayed a significant overall superiority on this instrument (t_{52}= 2.138, o^{3}diff. = .515, p. .05). The mean number of items correct for the fourth grade was 14.708; for the second grade, 13.607.

Performance on the individual items is presented for the two grades in terms of error frequencies in Table 1. Two items, nos. 4 and 7, appear to account for most of the descrimination between the two grade levels with No. 7 descriminating at a highly significant level ($X_1^2 = 17.875$, $p < .001$). It is interesting to note that both items 4 and 7 are of the transitive-asymmetric variety; evidently second graders are less able to use this relationship, particularly in the somewhat novel context of geographical inclusion.

It is interesting to note, also, that most of the subjects were able to identify correctly the symmetric, asymmetric, and transitive-symmetric relations which supports the findings of DeSoto (1958). He found, however, that children seem to perceive these relations less accurately than older subjects, offering support to the notion that maturational level does indeed play a role.

Since the present study involved such a small sample of the population, drawn entirely from a unique school setting (a laboratory school) where it could be expected that the children's introduction to new mathematics comes at an early age, it would seem desirable that a more extensive study be done involving children of other backgrounds. Although it seems obvious from these data that children of this age find the most difficulty with transitive-asymmetric relationships, further study with a larger and more diverse population should be carried out in order to support this observation conclusively. Future studies should also recognize the fact that verbal items involving these mathematical properties must attempt to control the background of the subjects so as not to contaminate correct recognitions with variations in relevant linguistic experience.

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Interaction of Visualization and General Reasoning Abilities with Curriculum Treatment in Algebra

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PURPOSE

This paper reports results of a study designed to test the following hypotheses:

> I. When scores on an immediate recall learning test following self-instruction on quadratic inequalities by either a graphical or an anlaytical treatment are regressed on measures of the aptitude variables General Reasoning and Visualization, the regression planes will not be parallel.
>
> II. When scores on a transfer measure following self-instruction on quadratic inequalities by either a graphical or an analytical treatment are regressed on measures of the aptitude variables General Reasoning and Visualization, the regression planes will not be parallel.

The theoretical argument in support of these hypotheses derives mainly from three bodies of literature; first, the literature supporting the importance of aptitude-treatment interaction studies in general; second, reports of research which relate spatial aptitude variables and mathematics achievement; and third, reports of research which is suggestive of a potential interaction between spatial aptitude variables and curriculum treatment in predicting mathematics achievement.

DESIGN

A statistical model appropriate for testing the stated hypotheses is known in the research literature as a test for homogeneity of regression. To make use of the model, it is necessary to obtain measures of the relevant aptitude variables for two randomly assigned experimental groups; subject the two groups to the distinct instructional treatments; obtain measures of learning and transfer following treatment; and regress the learning and transfer scores on the aptitude variables separately by groups. This procedure results in a regression equation for each group in which the aptitude scores predict either the learning test score or the transfer test score. An F-statistic is then computed to test for parallelism of the regression planes across treatment groups.

The independent variable for the experiment was the curriculum treatment varialbe. Concomitant variables were measures of Visualization and General Reasoning Ability. Learning test scores and transfer test scores were measures of the dependent variable for the study.

Procedure

Measures of Visualization and General Reasoning Ability were selected from Educational Testing Service's French Kit of Reference Tests for Cognitive Abilities. Two measures of visualization were selected, Paper Folding Test and Form Boards Test. Necessary Arithmetic Operations was chosen as a measure of General Reasoning.

Two instructional treatments of the topic quadratic inequalities were prepared in the form of constructed response linear programs. One treatment developed the topic in a graphical way, the other in an analytical manner. It was expected that learning and transfer would be facilitated by Visualization ability under the graphical treatment and by General Reasoning ability under the analytical treatment.

A ten item immediate recall learning test and an eight item transfer measure were developed and validated in terms of content. Five trial administrations of the instructional material and the criterion measures were performed to eliminate unexpected problems and to determine approximate timing.

Nine geometry classes were administered the aptitude measures yielding data for 228 Ss on the Necessary Arithmetic Operations test and the Paper Folding test. Ss were then randomly assigned to one of two treatment groups.

Two consecutive days were used for the self-instructional treatment groups.

Two consecutive days were used for the self-instructional treatment. Ss worked individually through the programs. Classroom teachers monitored the instructional period and collected booklets following each day's instruction. All Ss had adequate time to complete the instruction. On the third day, the criterion measures were administered. After losses during instruction and final testing, there were 84 subjects in the graphical treatment group with complete data and 97 subjects in the analytical treatment with complete data. Only these subjects were used in the analyses.

Results

In the case of hypothesis I (learning test), no evidence to support rejection of the null hypothesis was found. There was good evidence that both groups had learned from the instruction. The ability measures did not significantly predict performance on the learning test for either treatment, consequently no finding of interaction was possible.

The statistical test of hypothesis II. produced an F = 5.30, df: 2,175 which justified rejection of the null hypothesis with $p < .01$. This finding was confounded, however, by a criterion measure with an extremely low reliability. The unreliable nature of the transfer test cast strong doubt on the validity of the computed regression coefficients for the aptitude variables predicting transfer scores. It was, therefore, concluded that hypothesis II remains unconfirmed.

In order to provide greater insight into the relationship of the treatments and the aptitude variables, the statistical test of interaction was run using each item of the two criterion measures as a dependent variable. This analysis resulted in the discovery that two items of the transfer test were differentially predicted by the aptitude variables across treatments.

A secondary finding of this experiment, but one worthy of further study is that the two treatments, although very different in content development, were equally effective in producing learning and transfer averaged over levels of ability.

Implications for Further Research

This study lends additional evidence to support the existence of manageable aptitude-treatment interactions. The study should be replicated with reliable criterion measures.

There is a strong suggestion that the differentiation of instructional treatments used in this study could be applied to a wide range of mathematical content. This would allow extended treatment and a potential strengthening of the interaction effect.

The Relative Effectiveness of Four Strategies for Teaching Mathematical Concepts

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Purpose of the Study

One of the principal tasks of every mathematics teacher, as well as teachers of other subjects, is the teaching of concepts. The purpose of this study was to investigate the relative efficacy of four instructional strategies for promoting three levels of understanding of mathematical concepts.

The Conceptual Framework

An instructional strategy is a sequence of moves; a move used in teaching a concept is a segment of the dialogue carried on by a teacher, a student, or the teacher and one or more students. From a study of the research reported by Smith et al (3) and by Henderson (2) it was determined that there are basically two kinds of moves used in teaching concepts--characterization moves and exemplification moves. Characterization moves are those in which a person talks about the characteristics or properties of objects denoted by the term designating the concept. Exemplification moves are those moves in which members or nonmembers of the referent set are identified.

Four instructional strategies were identified on the basis of the kinds and sequences of the two major categories of moves employed.

1. Characterization--a sequence of characterization moves.
2. Characterization-exemplification--a sequence of characterization moves followed by a sequence of exemplification moves.
3. Exemplification-characterization--a sequence of exemplification moves followed by a sequence of characterization moves.
4. Exemplification-characterization-exemplification--exemplification moves followed by characterization moves followed by additional exemplification moves.

Since the attainment of concepts is not of a dichotomous nature, one of the objectives of the study was to observe the relationship between the efficacy of the four instructional strategies and the levels of awareness of the concepts attained by subjects. Three levels of awareness of concepts were identified. This classification of levels of understanding was based on the classical taxonomy of educational outcomes in the cognitive domain developed by a committee of college and university examiners and edited by Bloom (1). A modification of this classification was adopted in order to obtain a more general classification with fewer categories. This modification contains three gross divisions or levels. Level I emphasized knowledge and comprehension, Level II is concerned with application, and Level III pertains to analysis, synthesis, and evaluation.

Procedure

Four programmed booklets were prepared each using one of the four selected strategies. Each programmed booklet was designed to teach eleven selected concepts taken from elementary probability theory. A test was constructed, designed to measure the attainment of the concepts at each of the three levels of understanding.

Those students enrolled in a beginning college mathematics course who had had no previous training in probability were divided into upper and lower ability groups by using their scores on the mathematics section of the SAT. The students from each of the upper and lower ability groups were then randomly selected and assigned to the four treatments. This sampling procedure resulted in the selection of eight experimental groups containing 24 students each.

Scores reflecting the awareness of concepts at Level I, Level II, Level III, and a combination of Levels I, II, and III were each analyzed using a two-way analysis of variance design.

Findings

Using the .05 level of significance, a significant difference was found between the treatments when the awareness of the concepts was measured by Level I responses (the knowledge and comprehension level). Multiple comparisons of mean scores indicated that the characterization strategy, the instructional strategy consisting entirely of characterization moves, was more effective than either of the other three strategies for promoting the awareness of concepts at Level I.

No significant differences were found between the four instructional strategies when the awareness of the concepts was measured by Level II responses (the application level), Level III responses (the analysis, synthesis, and evaluation level), or a combination of Level I, Level II, and Level III responses. No statistical significant interaction between the treatments and levels of pupil ability was found for any of the levels of awareness tested. Significant differences were found between the two ability level means for all levels of awareness. This result was as expected on the basis of the criterion used in the selection of the ability groups. The .05 level of significance was used in all comparisons.

Conclusions

Several conjectures can be proposed to explain the relative efficacy of the four strategies. It is conceivable that the efficacy of various instructional strategies depends on such factors as the nature of the concept being taught, the level of understanding desired, and the age and intellectual development of the students to whom the concept is to be taught, among other factors.

For each strategy the number of moves was held constant. Each concept venture consisted of five moves. Perhaps different results would have been obtained if the number of moves required to attain a certain level of understanding had been used to measure the effectiveness of the strategies.

Another conjecture is based on the nature of characterization moves as compared to exemplification moves. It can be argued the characterization moves enabled the students to focus on those relevant aspects or details which are essential to understanding a given concept. In those strategies employing exemplification moves the student was expected to identify some of the properties associated with the concept from an examination of the exemplification moves.

Many experimental studies are needed in the area of concept formation because of the multitude of factors affecting the learning of concepts. The model of instructional strategies as sequences of characterization and exemplification moves provides a framework to design and conduct research in this area.

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The Acceptance and Recognition of Six Logical Inference Patterns by Secondary Students

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The study was an investigation of the responses of secondary (eighth, tenth and twelfth grade) students to six logical inference patterns presented under four contentent variations, without explicit training. The patterns selected were the law of detachment, contrapositive inference, hypothetical syllogism, disjunctive syllogism, denying the antecedent and affirming the consequent, all taken from sentential logic. The first four patterns are valid while the last two are invalid. These patterns are presented in symbolic form in Figure 1 to illustrate the nature of each.

Law of Detachment

If P then Q
P
Q

Contrapositive Inference

If P then Q
Not Q
Not P

Hypothetical Syllogism

If P then Q
If Q then Y
If P then Y

Affirming the Consequent

If P then Q
Q
P

Invalid Pattern
(The usual error) response

Denying the Antecedent

If P then Q
Not P
Not Q

Invalid Pattern
(The usual error) response

Disjunctive Syllogism

P or Q
Not P
Q

Figure 1

When the question of measuring the ability to accept or recognize logical inference patterns is considered, the question of item content immediately arises. Content is unimportant in the sense that the validity of a pattern is independent of content. However, there was evidence which indicated that the confusion of validity and perceived factual truth may be a common error. Four sources of content variation were identitied and considered in this study. The content variables used to measure each pattern were such that the premises as presented (a) satisfied familiar physical world situations, (b) violated familiar physical world situations, (c) were nonsense statements and (d) were symbolic statements.

Purposes

A. One purpose of the study was to determine which of the
   1) Four valid patterns were accepted as valid by the students in each grade.
   2) Two invalid patterns were recognized as invalid by the students in each grade.
   3) Two invalid patterns were accepted as valid by the students in each grade.

B. A second purpose was to investigate the differences in scores, if any, on the test and subtests as determined by pattern and content, when the subject variables were stratified on the basis of sex, grade level and ability level.

C. A third purpose was to construct a multiple choice response test which measures the acceptance or recognition of the six patterns.

Procedures

Four school systems were randomly selected from the 22 schools in southeastern Wisconsin which cooperate with Wisconsin State University-Whitewater in teacher training in mathematics.

To answer question A, 25 subjects were randomly selected from each of the three grades in each school system. An operational "mastery of pattern" was defined to determine the acceptance or recognition of a given pattern by an individual. An individual was said to accept a valid pattern as valid or recognize an invalid pattern as invalid if he responded correctly to 13 or more of its 20 instances in the test. Likewise, he accepts an invalid pattern as valid if he selects "the usual error response" to 13 or more of its 20 instances.

To answer question B, the students were stratified on the basis of sex, grade level (eighth, tenth and twelfth) and ability level. Three ability levels, as determined by the Henmon-Nelson Test of Mental Ability were used. The below-average ability group had IQ's below 101. The average ability group had IQ's between 101 and 113. The above-average ability group had IQ's above 113. This division separated the students so that 27 percent were below average, 46 percent were average and 27 percent were above average.

For the statistical analysis, a 2×3^2 factorial design, constituting 18 groups, was used. Twenty individuals were selected for each cell, making a total of 360 students in this sub-study. The 24 repeated measures, given by the six patterns and four contents, were separated into a 4 x 4 design involving the four valid patterns and a 2 x 4 design involving the two invalid patterns.

Results and Conclusions

The test which was developed for the study is a multiple choice test, containing 120 items and measures the acceptance or recognition of the six inference pattern. It consists of 24, five-item subtests. These subtests were determined by the six pattern and four content variables. The Hoyt Reliability Coefficients for the test were .91, .92, and .93 for grades eight, ten and twelve, respectively.

The study indicated that the majority of the students in each of the three grades accept both the valid and invalid patterns as valid. However, a few high-ability tenth and twelfth grade students did recognize the invalidity of the invalid patterns.

The analysis of variance indicated twelve sources of variation significant at or beyond the .05 level.

(1) The sex difference was significant at the .05 level, with girls scoring higher than boys; however, there were no significant sex interactions.

(2) Grade differences were significant at the .01 level, with the mean for the twelfth being higher than the mean for the tenth, which was higher than the mean for the eighth.

(3) Ability differences were significant at the .01 level with the order of means from greatest to lowest being above average, average and below average.

(4) Pattern differences were significant at the .01 level with the order of means from high to low being law of detachment, contrapositive inference, hypothetical syllogism and disjunctive syllogism.

(5) Content differences were significant at the .01 level with the order of means from highest to lowest being physical world, nonsense, violate physical world and symbolic.

(6) The other significant sources of variation were: grade by ability (.05), pattern by grade (.05), pattern by ability (.01), content by grade (.05), content by ability (.01), pattern by content (.01), and pattern by content by ability (.01).

The grade and ability interactions arose because the high ability eighth and tenth grade students scored about as well as the twelfth grade students on the test and subtests. However, there was a definite gradewise increase in scores from the eighth to the twelfth grade for the average and below average groups. The low ability eighth and tenth grade students also experienced greater difficulty in divorcing their cognitive processes from physical world situations than did other groups.

RESEARCH REPORT SESSION IV

FRIDAY

1:30 p.m.--2:30 p.m. Jackson/Lincoln/Roosevelt (L)

Presider: Edward M. Carroll, New York University, New York, N.Y.

Philip Gibbons, Southwestern State College, Weatherford, Oklahoma

A Comparative Analysis of the Impact of Various Methods of Instruction on Achievement and Understanding in Mathematics for Elementary Teachers

Anthony J. Picard, University of Hawaii, Honolulu, Hawaii

An Analysis of Achievement of Behavioral Objectives for a Freshman *Calculus Course*

J. Norman Wells, Georgia Southern College, Statesboro, Georgia

Incorporating Participation into a Startegy for Effectively Using a Dual-Media Instrument to Teach the Principle of Mathematical Induction

Jon L. Higgins, Stanford University, Stanford, California

Groupings of Student Attitude Attendant to a Mathematics Through Science Unit

A Comparative Analysis of the Impact of Various Methods of Instruction on Achievement and Understanding in Mathematics For Elementary Teachers

Philip Gibbons
Southwestern State College
Weatherford, Oklahoma

The careful preparation of prospective elementary teachers in mathematics subject matter is a prerequisite to an improved program in mathematics at the elementary school level. Today's elementary teachers must teach more mathematics, and do so in a more meaningful way, than have the elementary teachers of the past. Further, more elementary students go to high school and college than ever before; therefore, elementary teachers must be concerned with each student's understanding of mathematics as well as his computational skills. Elementary teachers, present and future, will not be able to meet present demands unless they are prepared in a more meaningful manner.

Therefore, the question of today's elementary teachers being fully prepared to teach today's elementary school mathematics is one that has been raised by many mathematicians and mathematics educators. There has been much written to support the fact that today's elementary teachers need to improve their basic knowledge and fundamental understanding of mathematics (2, p. 296), (10, p. 4), (28, p. 51). As a result of these many studies, stress is now being placed on the need to find ways to remedy the situation rather than the gathering of additional data to re-emphasize that elementary teachers are deficient in their mathematics preparation.

> Experimental studies should be undertaken in order to determine what content material and what types of presentation provide teachers with the knowledge and understanding that is most valuable to them as teachers of elementary school mathematics. They need competence much more than recall. (37, p. 398)

Therefore, the principal purpose of this study was to investigate potential ways to improve prospective elementary teachers' knowledge and understanding of elementary mathematics. A second purpose was to investigate whether or not the mastery of this mathematics was affected by the way it was taught at the undergraduate level.

The subject matter involved in the experiment is commonly referred to as modern mathematics for elementary teachers. Topics covered included set theory, the whole numbers, systems of numeration, fractions, the integers, the number line and its uses, and the rational numbers.

The research problem was designed to determine whether or not undergraduate classes that were exposed to a combination of programed learning, lecture, and discussion could achieve greater understanding in elementary mathematics than undergraduate classes that received only the lecture form of instruction.

The first experimental group, the Lecture Program Discussion grou! (L.P.D.) received the following method of instruction. Each new concept, or set of concepts, was first introduced through a lecture that was supplemented by a homework assignment that consisted of reading a certain number of frames from related programed materials. The concepts were then discussed in detail, by both students and instructor, at the next class meeting. This cycle was repeated throughout the entire course.

The second experimental group, the Program Lecture Discussion group

(P.L.D.) received the following method of instruction. Each new concept, or set of concepts, was first introduced through programed materials. The learner read these materials prior to attending a given lecture. These concepts were then supplemented and enlarged upon by a related lecture. Finally, the programed materials and the lecture were then discussed at the next class meeting. This cycle was repeated throughout the entire course.

The third experimental group, the Lecture Textbook group (L.T.) received the following method of instruction. Each new concept, or set of concepts, was introduced through a lecture. The assignment for the succeeding class was to solve a set of exercises from a related textbook. This method represented the traditional approach that has been and continues to be used at most colleges. There was no discussion unless a student requested the answer to, or an explanation of, a given exercise.

A control group, which received no instruction in elementary mathematics, was used in order to evaluate the effect of maturation.

Finally, if it had been determined that one of these elements in the teaching-learning process helped some students achieve significantly more than others, then a partial solution might be available for use by those interested in increasing the supply of mathematically competent elementary teachers.

Since no comprehensive theory of learning was available, the author attempted to develop one based on extensive review of available literature.*

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*The theory is too long to include in this outline. However if the paper is accepted, this theory will be explained in detail at the meeting.

The following hypotheses were deduced from the theory and rationale presented:

1. Those students involved in the L.P.D. method will show a significantly greater level of achievement and understanding in mathematics than those students involved in the P. L. D. method.

2. Those students involved in the L.P.D. method will show a significantly greater level of achievement and understanding in mathematics than those students involved in the L.T. method.

3. Those students involved in the P.L.D. method will show a significantly greater level of achievement and understanding than those students involved in the L.T. method.

Analysis and Conclusions

Each group was administered the pretest. *The Structure of the Number System* (Form A), during the first week of the semester in September, 1966. The posttest, *The Structure of the Number System* (Form B), was administered during the last week of the semester in January, 1967. The data that were used to test the hypotheses were the A.C.T. mathematics test scores, the pretest scores and the posttest scores.

Since there were significant differences between the groups on both the pretest scores and the A.C.T. mathematics scores, analysis of covariance was employed in comparing the groups on the posttest scores (44). Further analysis for comparing adjusted individual means used Tukey's procedure for comparing individual means (13, p. 330) which consisted of (i) testing for a significant gap, (ii) testing for a "straggler," and (iii) testing for excessive variability. This procedure allowed the author to draw as many conclusions as are reasonable about differences that were present among means.

The analysis of covariance disclosed the fact that significant didifferences existed among the four groups on the adjusted posttest results. Tukey's procedure indicated the following: (1) the control group mean was significantly smaller than the L.T. mean. The L.T. mean was significantly smaller than the L.P.D. mean and the P.L.D. mean. The P.L.D. mean was smaller than the L.P.D. mean, but not significantly smaller.

Based on these results, hypotheses (2) and (3) were accepted while hypotheses (1) was not accepted.

The most significant result of the experiment was the fact that Oklahoma State University has completely revamped their mathematics courses for elementary teachers. The procedure they are now using is that indicated by the L.P.D. method. The results to date are most gratifying.

Finally, Southwestern State College has made the same changes in their elementary mathematics courses and are also comtemplating the construction of an elementary school mathematics laboratory.

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An Analysis of Achievement of Behavioral Objectives for a Freshmen Calculus Course

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Introduction:

The rise of the behaviorist school of psychology led to the statement of educational objectives in terms of student behaviors. Current interest in behavioral objectives results from:

1) recent emphasis on teaching machines and programmed units.
2) attempts to develop curricula based on specific performance descriptions.
3) need to develop more valid and meaningful evaluation procedures.

Purpose:

The purpose of this study is to create a set of behavioral objectives for a sequence of calculus courses, to create a test designed to measure the achievement of these objectives, to measure achievement at various levels of the "Taxonomy of Educational Objectives" by Bloom and Krathwohl, and to compare student rating of objectives with faculty rating.

Procedure:

A collection of thirty-five proposed objectives for the first year sequence of calculus courses at the Ohio State University was generated using the "Taxonomy of Educational Objectives" by Bloom (cognitive domain) and Krathwohl (affective domain). Statements representing all levels of the taxonomy were included in this collection. Each statement was rated by selected members of the mathematics faculty as follows:

0) the statement is not an objective

1) the statement represents an objective of minor importance

2) the statement represents an objective of moderate importance

3) the statement represents an objective of major importance.

The arithmetic mean rating was computed for each statement. Test items were constructed for all statements with a rating r = 2.00 acceptable items were chosen by the faculty panel.

A preliminary test constructed from these items was administered to a group of fifty-seven calculus students at two of the university branches. The results were analyzed using the Ohio State University Item Analysis Program. This analysis provided mean and median scores, standard deviation, reliability as computed by Kuder-Richardson Formula #20 and Formula #21, and reliability as computed by an odd-even split. It also provided a coefficient of difficulty and a coefficient of discrimination for each item of the test. Items which were not difficult and did not discriminate were modified or replaced.

The revised test was administered to a group of one hundred ninety-three students concluding the first year calculus sequence at the main campus of the Ohio State University. The students also rated the objectives using the rating scale employed by the faculty panel and the arithmetic mean rating was computed for each statement.

Analysis of Results:

The results of the entire test were analyzed using the Ohio State University Item Analysis Program described above. The test items related to objectives at level 1 (Recall), level 2 (Comprehension), and level 3 (Application) of the Cognitive Domain of Bloom's Taxonomy were grouped into subtests. The Item Analysis Program was used to analyze the results of each of these subtests. There were insufficient items for the upper levels of the cognitive domain and for the affective domain to permit similar analyses.

The lowest reliability for the entire test was .77; for the subtest related to level 1 (Recall) .77; for the subtest related to level 2 (Comprehension) .62; for the subtest related to level 3 (Application) .17. The rank difference correlation coefficient between the faculty and student ratings was computed as .74. This is significant at the .01 level.

The percentage of incorrect responses for each item was used as the criteria for achievement of the objective. This was computed by dividing the number of incorrect responses by the total number of responses and converting this figure into a percentage. A mean percentage of failure was computed for each objective and for each cognitive level.

The group of objectives associated with level 1 was achieved more satisfactorily than the group associated with level 2 or 3. The objective achieved most successfully at this level stated, "The student should identify all the symbols used in the first year calculus, e.g., "$\frac{dg}{dx}$, f, $\int f$, $\int_a^b f(x)dx$."

The achievement of the group of objectives at level 2 was comparable to the achievement of the group of objectives at level 3. The number of items related to level 3 objectives was smaller than the number related to level 2 objectives and hence the reliability of subtest three was lower than the reliability of subtest two. The objective achieved most successfully at level 2 stated, "The student should define intuitively the technical terms of the first year calculus, e.g., the limit of a function, continuity, derived function, critical point, definite integral." The objective achieved most successfully at level 3 stated, "The student should compute the derivative and the definite integral of common functions, logorithmic functions, and expoential functions."

Although there was no significant difference in the ranking of the set of thirty-five objectives by faculty and students, there were instances in which the mean rank of individual statements differed considerable. For example, "The student should be willing to expand mathematical knowledge by independent reading" was ranked 18 by the faculty and 30.5 by the students. The statement, "The student should isolate the logical structure underlying a proof" was ranked 17 by the faculty and 2 by the students.

Conclusions:

The analysis of data implies student achievement is highest at the lowest level of Bloom's Taxonomy. This supports similar research involving the taxonomy. Many statements relating to higher level cognitive processes and to the affective domain received low ratings by the faculty panel implying that the faculty conceive of the first year calculus sequence as basically fact oriented.

The students received no direct information on the objectives of the calculus courses. The high correlation between faculty ranking and student ranking suggests that the students were able to obtain useful information on the objectives from indirect sources such as examinations and emphasis on presentation. The high correlation in rank order also implies that the students perceive the first year calculus sequence as basically fact oriented.

The objectives with the greatest difference in faculty-student ranking can be grouped into two categories, outside readings and logical structure. The faculty assigned a much higher ranking (and hence a much more important function) to objectives indicating an ability and a desire to read mathematics independently. The students ranked objectives relating to logical structure (of a proof and of calculus) and deduction much higher than the faculty. The students seemed to feel that deduction and logic played a more dominant role than the faculty considered necessary or important.

Incorporating Participation Into A Strategy For Effectively Using A Dual-Media Instrument to Teach The Principal of Mathematical Induction

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Purpose

Some educators have suggested that filmed instruction cannot only help solve the problem of increasing enrollments and teacher shortages, but also provide instructional improvement. As a result of this encouragement, film producers and mathematical organizations have increased their production of films for mathematical instruction.

The likelihood of increased production and employment of mathematical films makes it imperative that the mathematics educator seek ways to use these films more effectively. Previous film research has demonstrated that learning from a factual film increases with the use of audience participation procedures. The film research to date, however, has not involved the participation variable in situations which require the learning of mathematical tasks. This question arises: Which results of previous research can be applied to mathematical learning? And, in a mathematical learning situation, what form should the participation take?

The effectiveness of certain participation techniques reported in film research studies led the investigator to hypothesize that a programed text could prove to be an effective method of obtaining student response during the showing of a mathematical film. The investigator furhter hypothesized that a theory proposed by Robert M. Gagne would serve as a basis for determining both the content of the program and the times during the film showing when the program should be studied.

The main purpose of the investigation was to contrast the achievements of two experimental groups (one using the programed text as a during film participation device, and the second using the program as a pre- and post-film study device) with that of a control group who simply viewed the film. Additionally, the relative effectiveness of the three treatments for subjects at different ability levels was determined.

Research Hypothesis and Related Questions

The following research hypothesis was tested in this study:

> A dual-media instrument, incorporating a mathematical film and a programed text, is more effective when the program is used as a during film participation device than when the program is used as a pre- and post-film study device.

Here "more effective" refers to performance on a common performance test designed to measure the achievement of subjects who used the instructional instruments in each of the two settings. The null hypothesis is:

> Performance test means are the same for subjects who use dual-media participation instrument as for those who use the dual-media non-participation instrument.

While the research hypothesis thus states the basic problem considered in this study, other subordinate questions were investigated to some extent. These are:

1. How do students from each treatment group compare in their retention of knowledge acquired?
2. Are the methods of presentation equally effective for all students? More specifically, do students of a higher or lower ability level respond more favorably to one or another presentation method?

3. What is the comparison in mean times required to complete each presentation method?

4. How do the two treatments, dual-media participation treatment and dual-media non-participation treatment compare with a film-only presentation in effectively teaching a complex mathematical task?

Method

Analysis of Final Task

The final task required of the subjects--Proving mathematical statements using the principle of mathematical induction--was analyzed by the author and 22 subordinate tasks were identified. These tasks were organized into a hierarchy of tasks following a pattern used by Robert M. Gagne.

Matherials

The Film

The mathematical Association of America (MAA) film entitled Mathematical Induction--A Lecture of Leon Henkin was used in this study. Professor Leon Henkin, a member of the Committee on Educational Media of the MAA, is well regarded as a lecturer and is an authority on the PMI and related topics. The film is approximately one hour in duration.

The Programed Text

The programed text was written by the investigator. The content of the program was based on two factors: (1) the learning sets represented in the hierarchy, (2) the adequacy of treatment given in the film for certain of these learning sets. The program segments used during the film showing are referred to as subprograms.

Tests

Three different tests (Pre-, Performance, Retention) were employed to assess the subject's knowledge of the final and subordinate tasks defined in this study. The Pretest was given the day before instruction began. The Performance Test was administered on the day following instruction. The Retention Test was given two weeks after the instruction was completed.

Each of the tests contained thirty-three items. Items 1-29 measured achievement on each of the subordiante tasks defined in the hierarchy. Items 30-33 were typical of induction exercises usually encountered in contemporary college freshman algebra texts. Each of the items 30-33 were typical of induction exercises usually encountered in contemporary college freshman algebra texts. Each of the items 30-33 was weighted 50 points, giving a maximum possible score of 200 points. The scores on items 30-33 of each of the three tests and the time spent in studying the programed text served as criteria measures.

Subjects and Treatment Groups

The 179 students participating in this study were enrolled in either of two freshman mathematics courses offered at Georgia Southern College, Statesboro, Georgia, during the fall quarter of 1967. All students with scores less than 440 (score scale of 200-800) on the mathematics section of the CEEB Scholastic Aptitude Test were grouped as one ability level and all with scores above 440 served as a second ability level.

Using the film and subprograms, two dual-media instruments were designed to test the research hypothesis and answer related questions. One instrument provided for a presentation which did not incorporate participation during the film showing. Subjects using this instrument are referred to as the Experimental One (E-1) group. A second group, the Experimental Two (E-2) group, used an instrument which incorporated student participation during the film showing. A third group, the Control group, simply viewed the film.

Procedure

The experiment was conducted following the schedule shown in Table 1.

Table1.--The schedule of the experiment

Day	Treatment Group		
	E-1	E-2	Control
Tues., Oct. 24, '67	Pretest	Pretest	Instruction
Wed., Oct. 25, '67	I[a]	I	Instruction
Thurs., Oct. 26, '67	II-III	1[b]-II-2-III	Instruction
Fri., Oct. 27, '67	IV-V	3-IV-4-V	Instruction
Mon., Oct. 30, '67	Film	5-VI	Instruction
Tues., Oct. 31, '67	VI	6-VII	Pretest
Wed., Nov. 1, '67	VII-VIII	7-VIII	Film
Thurs., Nov. 2, '67	Perf. Test	Perf. Test	Perf. Test
Thurs., Nov. 16, '67	Ret. Test	Ret. Test	Ret. Test

[a] Roman numerals refer to the subprograms.

[b] Hindu-Arabic numerals refer to the film sequences.

[c] The instruction was on a unit in probability (unrelated to film).

Results

In comparing the performance (immediate and delayed) of the groups using the dual-media instruments, the performance and retention test data were analyzed using a 2 x 2 (treatment by level) factorial design. The analyses of variance (Tables 2 and 3) for these data indicated no significant treatment effects and no significant treatment-ability level interaction effects. The analyses did indicate significant (at the .05 level) ability level effects.

Table 2.--Summary of analysis of variance of Performance Test scores for groups E-1 and E-2 (levels 440 and $\overline{440}$)

Source of Variation	SS	df	MS	F
Treatments	1,765.28	1	1,765.28	0.79
Levels	41,157.88	1	41,157.88	18.49
Treatment x Levels	973.23	1	973.23	0.44
Within Cell	233,617.35	105	2,224,93	
Totals	277,513.74	108		

Table 3.--Summary of analysis of variance of Retention Test scores for groups E-1 and E-2 (levels 440 and 440)

Source of Variation	SS	df	MS	F
Treatments	295.89	1	295.89	0.13
Levels	66,354.64	1	66,354.64	28.85
Treatment x Levels	4,389.86	1	4,389.86	1.91
Within Cell	241,552.85	105	2,300.50	
Totals	312,593.23	108		

The two dual-media treatment groups were further compared with the criterion variable being time spent in studying the programed text. The analysis of variance (Table 4) indicated a significant treatment effect, i.e., the non-participation group took significantly (.05 level) longer to study the programed text than the participation group. The ability level and interaction effects were not significant.

Table 4.--Summary of analysis of variance of times spent in studying the eight subprograms for groups E-1 and E-2

Source of Variation	SS	df	MS	F
Treatments	51,083.76	1	51,083.76	33.33
Levels	56.46	1	56.46	0.04
Treatment x Levels	2,563.21	1	2,536.21	1.67
Within Cell	160,939.48	105	1,532.76	
Totals	214,642.91	108		

In contrasting the performance (immediate and delayed) of the groups using the dual-media instruments with the groups who simply viewed the film, the analyses (Tables 5 and 6) indicated that either of the dual-media groups performed significantly (.05 level) better than the film-only group. These analyses also indicated that higher ability subjects, regardless of treatment, performed significantly (.05 level) better than lower ability subjects.

Table 5.--Summary of analysis of variance of transformed[a] Performance Test scores for groups E-1, E-2, and control

Source of Variation	SS	df	MS	F
Treatments	10.312	2	5.156	70.43
Levels	1.545	1	1.545	21.11
Treatments x Levels	0.162	2	0.081	1.10
Within Cell	11.126	152	0.073	
Totals	23.145	157		

[a] Due to nonhomogeneity of variances the scores were transformed using the transformation $x' = \arcsin\sqrt{x}$

There was also a significant (.05 level) treatment-ability level interaction effect on delayed performance (as measured by the Retention Test).

Table 6.--Summary of analysis of variance of transformed[a] Retention Test scores for groups E-1, E-2, and control

Source of Variation	SS	df	MS	F
Treatments	6.674	2	3.337	39.61
Levels	2.111	1	2.111	25.06
Treatments x Levels	0.650	2	0.325	3.85
Within Cell	12.803	152	0.084	
Totals	22.238	157		

[a] Due to nonhomogeneity of variances the scores were transformed using the transformation $x' = \arcsin\sqrt{x}$.

Conclusions

The principal conclusions from this study were:

1. Films designed for mathematics instruction might be more effective is used in combination with written programed material.

2. Results obtained from previous film research indicating the value of participation cannot automatically be transferred to films which teach mathematics.

3. If the expenditure of instructional time is a factor of consideration, a written program, when combined with a film presentation, can be better used as a participation device than as a pre- and post-film study device.

Groupings of Student Attitudes Attendant to a Mathematics Through Science Unit

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Most mathematics educators agree on the importance of using physical materials to approach mathematical abstractions in the early elementary grades. However, whether or not the use of such an approach should be continued in the middle and upper grades is currently an open question. School Mathematics Study Group has written three sample units suggested for grades seven, eight, and nine which utilize such an approach. The three units, designated Mathematics Through Science, deal with Measurement and Graphing; Graphing, Equations and Linear Functions; and An Experimental Approach to Function. Informal feedback during the pilot-testing phase of the development (prior to final revision) indicated an extremely mixed reaction to this approach on the part of both teachers and students.

In the spring of 1968, this researcher, supported by School Mathematics Study Group, undertook a more formal evaluation of student reactions to one of these units (Graphing, Equations and Linear Functions). The major intent of this evaluation was to explore changes in student responses to variables in the affective domain during the study of the unit. Twenty-nine eighth grade teachers from junior high schools in Santa Clara County, California were selected to teach the unit to one of their eighth grade mathematics classes. Both text materials and laboratory equipment were furnished to the schools by School Mathematics STudy Group. In addition, four inservice meetings were held for the teachers in order to acquaint them with the materials and to give them the opportunity to investigate the use of the laboratory equipment themselves before presenting it to students. Teachers were given four weeks in which to teach the unit, exclusive of time spent on testing.

A test battery was administered immediately before and after the four week instructional period. The battery consisted of three achievement scales and eighteen attitudinal scales selected from the tests developed for the National Longitudinal Study of Mathematical Abilities. The achievement scales used were "Algebra Number Properties," "Algebra Sentences," and "Algebra Translation."[1] Mean scores for the total student population showed significant gains from pre-treatment to post-treatment for all three achievement scales when compared by means of a t-test judged at the $p < .001$ level.

Pre- and post-treatment comparisons were also made by means of a t-test for all 18 attitudinal scale pairs. Significant differences at the $p < .01$ level were found for the scales "Mathematics vs. Non-Mathematics," "I think Father Uses Mathematics on the Job," and "I would Like to Take More Mathematics." In addition, significant differences at the $p < .001$ level were found for the scales "Mathematics Fun vs. Dull," "Ideal Mathematics Self-Concept," and "Acutal Mathematics Self-Concept." For five of these six comparisons mean scores were lower for post-treatment scores than for pre=treatment scores. The exception to this trend was the scale

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1. For detailed scale descriptions and statistics, see NLSMA Reports, No. 1-6, School Mathematics Study Group.

"I Think Father Uses Mathematics on the Job," which showed a gain in mean score.

Because previous evaluation during the pilot testing had indicated that some groups of students responded differently towards the materials than other groups, it was decided to analyze the attitudinal test data for the presence of naturally-occurring groups. The statistical method chosen for this analysis was Hierarchical Grouping Analysis.[2] This program considers a profile vector of test scores for each individual, and begins by considering each individual as a "group." These groups are then reduced in number by a series of step decisions. At each step, some pair of groups is combined, the selection being made so that the total within-groups variance is minimally increased. This increase is printed out as an error term at each step, so that in practice the researcher determines the final number of groups by deciding on the maximum increase in the error term which he will accept between successive steps.

Because Hierarchical Grouping Analysis is not necessarily predictive, the procedure was carried out for two samples of the population. Each sample was formed by randomly selecting four students from each teacher's class. Components of the individual profile vectors used as the basis for grouping were the six changes in pre- and post-treatment scores for each of the six attitude scales which had shown significant differences between pre- and post-treatment administrations.

The grouping procedure for each sample showed fairly regular in-

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2. See Donald J. Veldman, Fortran Programming for the Behavioral Sciences. New York: Holt, Rinehart and Winston, 1967.

creases in the error terms associated with each grouping step until the step which reduced eight groups into seven. The increase in error for this step was about three times the increase associated with the previous step for one of our samples; for the other sample it was about fifty times the increase associated with the previous step. Consequently, we terminated the procedure with the formation of eight student groups within each sample.

At this point the performance of the eight groups on each of the six pre-treatment attitude scales involved in the profile vector was analyzed using standard analysis of variance techniques. No significant differences between the eight groups on any of these six scales was found for either sample, supporting our use of score changes in the original profile vectors. Similar analysis of variance among each of the eight groups of both samples was run for the remaining twelve pre-treatment and twelve post-treatment attitude scales, as well as all achievement scales. But the interpretations of these analyses hinge on the identification of the eight groups.

Identification and description of the groups by means of interpretation of their mean profile vectors is easier for some of the eight groups than for others. It is easy to identify one group in each of the samples whose attitudes change favorably towards mathematics on all six components of the profile vector. This favorable group accounts for 7 percent of the population of one sample and for 8 percent of the other. It is also relatively easy to identify one group in each of the samples whose attitude changes are unfavorable on all six components.

This unfavorable group accounts for 3 percent and 8 percent of the sample populations respectively.

Each of the samples contains one group where changes appear to be minimal on all six components. This group accounts for 11 percent and 36 percent of the respective sample populations. An additional group with minimal changes on all but the "I Would Like to Take More Mathematics" component appears in each sample. This group accounts for 49 percent and 12 percent of the sample populations, respectively.

The four remaining groups in each of the samples show favorable changes on some components and unfavorable changes on others; and interpretations of these groups, while possible, are much more difficult to justify. For example, one group of each of the samples is characterized primarily by a gain on the "Mathematics vs. Non-Mathematics" scale and a drop in the "Actual Mathematics Self-Concept" scale. A possible description of this group is that they see this approach to mathematics as more interesting, but feel they can do less well at it. This group accounts for 3 percent and 9 percent of the sample populations, respectively.

The largest groups of both samples are notable mainly for their lack of dramatic attitude changes. While both favorable and unfavorable groups can be identified, the population of these groups is relatively small and is approximated by each of four additional groups in which students show varying types of ambivalence toward the use of physical materials in the teaching of mathematics.

Thus it would seem that, in general, strong cohesive attitude changes among students are not a factor for major consideration in developing or adopting mathematics taught via physical materials at the middle grades.

Effect of Precise Verbalization on Transfer

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PURPOSE

The purpose of this experiment is to provide evidence concerning the effect of precise verbalization of discovered mathematical generalizations on transfer ability.

BACKGROUND

Mathematics educators are essentially agreed that the student should discover as many generalizations as possible, but opinions differ on the advisability of immediate verbalization of these discoveries.

Hendrix and others have advocated that the teacher not have the student immediately attempt verbalization of a discovered mathematical generalization on essentially two grounds:

1. There is common evidence that a student does not have the linguistic capacity to state his discovery with precision. The verbalization is quite likely to be inadequate or incorrect, with, potentially, undesirable effects.
2. There is research evidence that a student who immediately attempts to state his discovery is less able to use that discovery than one who possesses the discovery on a non-verbal awareness level.

On the other hand, Ausubel argues that the verbalization of a subverbal insight is an integral part of the thinking process; he would not have teachers leave a discovery in a non-verbal awareness level because this would be an abortion of the thinking process.

Henderson pointed out that ultimately the teacher encourages the student to verbalize his discoveries and that the length of delay before verbalizing essentially depends on the student's facility with language.

Retzer, using a college capable and gifted population, provided research evidence that tended to weaken the first of Hendrix' arguments. This aspect of the experiment was replicated as Phase I of a two-phase experiment supported jointly by Illinois State University and Region Five of the Department of Health, Education and Welfare as Project No. 8-E-019. A report of Phase I was made at the 1968 National NCTM Convention. For a college capable and gifted research population, as well as for a normally distributed population, studying some selected concepts of logic enabled the treatment group to verbalize discoveries with more precision.

If the results of these experiments were generalized, a practical effect might be that a mathematics teacher, who felt that verbalization of a discovered mathematical generalization should be delayed on the basis that the student does not have linguistic capacity to state his discovery precisely, might choose to incorporate certain concepts of logic which tend to produce increased linguistic facility as an explicit and integral part of the mathematics curriculum. However, the teacher might still prefer to delay verbalization, in spite of his apparent ability to increase the linguistic capacity of the student, as a result of research evidence that a student who immediately attempts to state his discovery is less able to use that discovery than one who possesses the discovery on a non-verbal awareness level. The proposed research report would share current findings on this aspect of delaying verbalization of mathematical discoveries and would constitute a report of Phase II of research described in the August, 1968 NCTM Research Newsletter.

THE PROBLEM

Hendrix has stated that as far as transfer power is concerned, the whole thing is there as soon as the non-verbal awareness has dawned. Verbalization of the discovery seemed to diminish the power of some persons to apply the generalization.

One may question if the loss of transfer power which was concomitant with imprecise verbalization would also be experienced if the student could state his discovery with precision. In a bulletin devoted to research in mathematics education, Hendrix places his question on a list of critical unanswered questions, "Would immediate linguistic formulation of an individual's discoveries have the same detrimental effects if he had sufficient linguistic power to avoid making incorrect trial sentences?" Becker and McLeod summarized the current (1967) state of research in this area and related areas, " . . . the role which verbalization plays in transfer of mathematics learning remains unclear. Consequently specific additional research is needed in these areas."

The purpose of this experiment, then, is to provide evidence concerning whether the loss in transfer power, observed in other research, would still be a problem for those students who can verbalize with precision; in other words, it provides evidence concerning whether an improvement in verbalization ability will be followed by an improved ability to use the discovered generalization. Thus the objective of this research is to test the effect of an ability to verbalize discovered mathematical generalizations upon the ability to use those generalizations.

PROCEDURES

Subjects were assigned to groups according to their ability to verbalize as demonstrated by tests administered in Phase I of this experiment. Each subject in the research population completed a programmed unit on exponents denoted B, C. or else D. B, C, and D were designed to lead the student to discover generalizations about exponents. A text of the ability to use the discovered generalizations was administered.

DESIGN

The research design used in the experiment was a three-by-two analysis of variance; the first factor has three levels and the second, two. The dependent variable is the ability to use mathematical generalizations which have been discovered. The independent variables are the following factors listed with their respective levels:

Factor A : Verbalization of discovered generalizations or lack thereof.

A_1: Having completed exponent program B which contains no verbalization of discoveries.

A_2: Having completed exponent program C in which the text verbalizes the generalizations after the student discovers them.

A_3: Having completed exponent program D in which verbalization of the discoveries is elicited from the students.

Factor B : Ability to verbalize discovered generalizations with precision.

B_1: Above the median score for precision of verbalization of the generalizations discovered in Phase I of this experiment.

B_2: Below the median score for precision of verbalization of the generalizations discovered in Phase I.

The hypotheses tested in this experiment follow:

H_1: Verbalization of discovered mathematical generalizations has no effect on the ability of junior high school students to use the generalizations.

H_2: The ability to state discovered mathematical generalizations with precision has no effect on the ability of junior high school students to use the generalizations.

H_3: The effect of verbalizating discovered mathematical generalizations on ability of junior high school students to use the generalizations is independent of the ability to state the generalizations with precision.

Ten students were assigned to each cell of this research design, thus the main effect of Factor A was tested with three groups of twenty subjects each and the main effect of Factor B was tested with two groups of thirty.